Lift during wing upstroke

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1. Introduction

The basic principle of lift and thrust generation in flapping flight has already been described by Erich von Holst¹, 1943. In his functional scheme (see following Figure 1) the location of the centre of lift is represented by a wing section which is shift able along the half span of the wing. On the top of the stroke it is shifted towards the wing tip and at the bottom point to the wing root. In this way, seen over a whole flapping period, while maintaining the lift force F_L the thrust F_T on downstroke can get larger than the backward directed additional drag - F_T on upstroke.

This means that also on upstroke the lift can be about of the same size as in gliding. At the same time the upstroke plays an important role in the generation of thrust of the whole flapping period, even if it self altogether does not generate positive thrust. For an optimal design of the wing upstroke is necessary a concentration of the lift in the mid-span. The related, technical relationships how it can happen at least approximately will be described here.

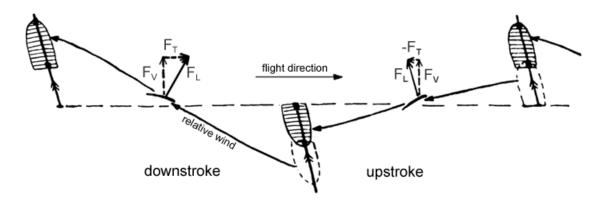


Figure 1. Basic principle of lift and thrust generation by lift displacement in the flight of birds, by Erich von Holst¹, 1943

In the research of the bird flight has always been discussed whether the wing upstroke happened, with muscle strength or aerodynamic forces. To clarify the corresponding physical processes, first with a technical flapping wing a theoretically experiment is executed here. For this, a wing on its wing root is rotatable mounted in a wind tunnel (see following Figure 2). With airflow from the front and positive angle of attack along the whole span is lift developed by the wing. If it is big enough, the wing tip will be raised.

During the turn upwards especially the outer wing area is blown more from above. The angle of attack there is getting smaller or even negative. For a strong power development this is not ideal. To compensate this effect, the wing will be twisted beginning from the wing root. The

angle of incidence at the wing tip will be increased so that the angle of attack is positive also during rotary motion of the wing. Thereby it is advantageous a permanent adjustment to the speed of rotation.

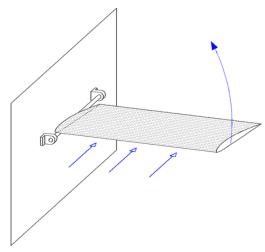


Figure 2. A wing at the root rotatable mounted in a wind tunnel

One can now also arrange the wing rotatable freely. Then it will not only flap up, but rotate continuously around its axis. The rotating wing can drive a generator. It then works like the blade of a wind turbine and emits energy. To remember:

lift force [N] x pressure point distance from the wing root [m] = torque [Nm] torque [Nm] x angle of rotation [rad] = work or energy [Nm]

The energy which the wing gives up to the generator is detracted before by an additional drag from the air flow. In the wind tunnel the airflow is decelerated. In contrast, in free flight it is the mass of the aircraft whose speed is reduced.

The upstroke of a flapping wing can function in the same manner. However, it is known that the downstroke works like a propeller. That now the upstroke shall act as a wind turbine, therefore initially seems to be paradoxical. There the upstroke would negate the effect of the downstroke. However, from the following Figure 3 you can easily read that on upstroke in the range with positive angle of attack can be generated lift and on negative angle of attack also thrust. Both are positive properties. So, it depends on the details. The big advantage of an upstroke with the function as a wind turbine is the lift which is developed thereby. If no lift is generated on upstroke, the whole lift of the aircraft must be generated only on downstroke.

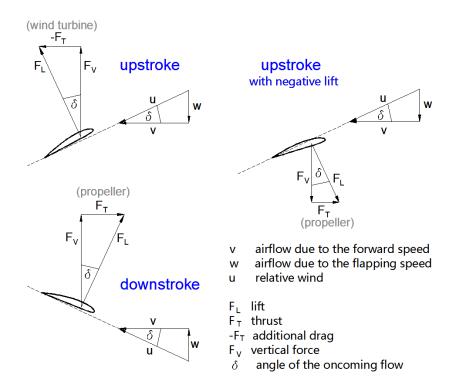


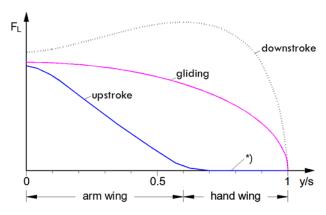
Figure 3. Forces on the flapping wing in the outside wing area

This is an illustration without profile and induced drag.

One can also term the additional drag –F_T during the upstroke with positive lift as operating or working drag of the wind turbine function.

When applying the designations of the forces in Figure 3 there is an anomaly. It appears in particular because "thrust" and "additional drag" named the same physical quantity. It is always the same component of the lift force. However, it changes at the zero crossing of upstroke to downstroke or on upstroke along the span not its sign but its designation. This is misleading and it comes to misunderstandings about the source of the additional drag. It is nothing more than thrust against the direction of flight. But the change of designation is used by biologists in birds, on modelling in ornithopters and also here. However, it is necessary to term this physical quantity at least in calculations throughout as thrust and then to accept the change of sign.

In the commonly used theory of bird flight mostly is assumed that during wing upstroke in cruise flight the hand wing is guided upwards without significant force generation (see Figure 4).



*) This is only possible, if the flow along the wing is neglected

Figure 4. Approximate course of the lift distributions according to the commonly used theory of the cruise flight of birds.

The lift distribution of the downstroke shown here, has already been used by J. Rayner² in his describtion of bird flight.

Also in ornithopters with straight wings one imagines the wing upstroke in the hand wing area about like this.

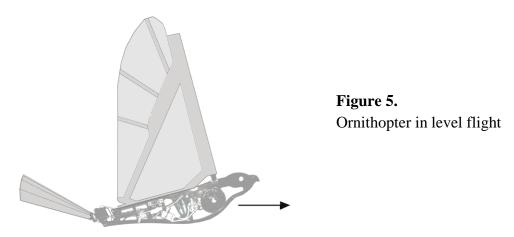
However, the arm wing alone cannot generate much lift. Also, nothing is reported from an increase in the angle of attack or other lift-enhancing measures of the arm wing. Accordingly, the lift of birds during upstroke would be much smaller than in gliding flight. During downstroke, on the other hand, should be generated most of the lift. The disadvantages of this way of flight are identifiable in today's ornithopters.

Birds are often admired because of their lightweight construction. This applies, for example, to hollow bones, feather-weight feathers, air sacs in the body and for various other biological features. In contrast, birds as a whole have a relatively high weight, at least from the point of view of an aero modeler. Current ornithopters, however, are usually very light. In the following Table 1 are specified some examples.

Ornithopter	wing span [m]	weight [kg]	wing loa- ding [N/m ²]	Birds (H. Tennekes ³)	wing span [m]	weight [kg]	wing load- ing [N/m ²]
Codeind				Comion Cuore		-	
Cybird	0.9	0.29	16	Carrion Crow	0.8	0.6	46
Park Hawk 1	1.2	0.43	17	Peregrine Falcon	1.1	0.8	62
Slow Hawk 2	1.2	0.44	13	Herring Gull	1.4	1.1	52
SmartBird	2.0	0.45	9	White Stork	1.9	3.1	61
				Greylag Goose	1.6	3.2	115
				Mute Swan	2.4	11.8	170

Table 1. Comparison of the flight weights and wing loadings

Although to days ornithopters have very powerful drives, nevertheless they hardly tolerate a payload. A fuselage fairing to protect the drive mechanism is often too heavy for them. This lift weakness will be counteracting somewhat by large depth of the wing root (similar to Flying Foxes or bats) and by strong erecting of the fuselage (see Figure 5). Thereby, will be increased particularly the angle of attack in the wing area close to the fuselage and so the lift there. In addition, by the inclination of the stroke plane the thrust will be directed a little upwards at the same time and therewith replaced missing lift. The power requirement of this flight is considerable.



Thus, the configuration of the wing upstroke has certainly a substantial influence on the power consumption and the load-carrying capacity of ornithopters.

2. Operating modes of the wing upstroke

A basic upstroke lift distribution is shown in the following Figure 6. In the area close to the fuselage the lift is positive. With its motion in the direction of the lift force the flapping wing works there as a wind turbine. Thereby generates an additional drag against the flight direction. In the remaining area near the wing tip the lift is negative. In this way the flapping wing acts there like a propeller and generates thrust.

The flapping motion near the wing tip is significantly larger than near the wing root. Accordingly, also behaves the respective performed work along the half span. The smaller area with negative lift near the wing tip here performs the same work during rotary motion as the larger area with positive lift near the wing root. Additional drag and thrust in this special case just equate each other. Also, the opposite torques of positive lift and negative lift are exactly the same. Thus, the wing can be moved upwards without an external force.

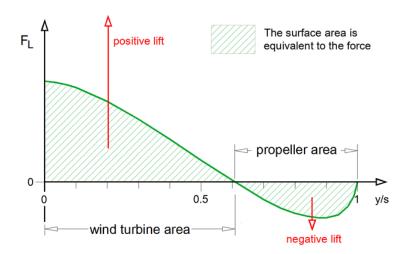


Figure 6. Lift distribution on upstroke with balanced torque and balanced thrust It marks the boundary between predominantly propeller operating mode and predominantly wind turbine operating mode.

The pressure point of this distribution lies directly on the wing root.

Calculated with the computer program "Orni 1"4

 $F_L = lift force$

y/s = relative half span

From the graph you can read an important generally applicable principle for flapping wings. Wing sections with the lift force in the direction of motion operate as a wind turbine and wing sections with the lift force against the motion operate as a propeller. At the same time, one of the ways is recognizable in this figure how can be used the wind turbine energy which is generated in the inner wing area. It can be used directly for thrust generation in the outer wing area. The inner wing area thereby drives the outer wing area in upstroke direction. This is probably the most important method for recovering of wind turbine energy in flapping wing upstroke. It is certainly also be used by birds.

The somewhat surprising of this lift distribution is, that the without torque upward moving flapping wing is still producing some lift (27% of the lift in gliding flight with elliptical distribution). The upward directed force of the wind turbine range is much bigger than the downward directed force of the propeller range. This confirms an important feature of the wing upstroke. Also, at it can be generated lift without additional loss of energy.

For most current ornithopters it is basically difficult to generate significant lift during the upstroke. Their motor is in full operation not only on downstroke but also on upstroke. That's about the way of flying of hummingbirds in stationary flight. Only the wing thereby can absorb the energy output of the motor. To develop an equal opposite force, the wing is

constrained to work with a large propeller area also on upstroke. Indeed, thereby is developed very much thrust but instead less lift. The lift distributions of ornithopters which are flying in this way then will look like in Figure 7.

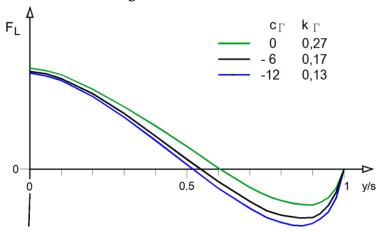


Figure 7. These are lift distributions for the upstroke in propeller mode. For comparison is also shown the lift distribution with balanced torque, with distribution parameter respectively circulation characteristic number (c-Gamma) $c_{\Gamma} = 0$.

The circulation factor \mathbf{k}_Γ (k- Gamma) describes the size of lift based on that of gliding flight.

 F_L = lift force y/s = relative half span

The areas with positive and negative lift are relative strongly developed and lies directly side by side. Thus, the induced drag is large. The resulting total lift, however, is very small. Excess wind turbine energy is not available. On contrary, the thrust generation predominates and is considerable on such an upstroke. But the profile in the outer wing area thereby must work during upstroke with strong negative and during the downstroke with strong positive angles of attack. This is almost only possible with membrane wings. They can camber their profile form flexibly upward and downward.

The following Figure 8 shows some lift distributions for the upstroke with significant lift generation. They range from the lift distribution with balanced torque to a lift distribution with a throughout positive lift. Thus, these lift distributions cover about the operating range of a wing upstroke in wind turbine mode.

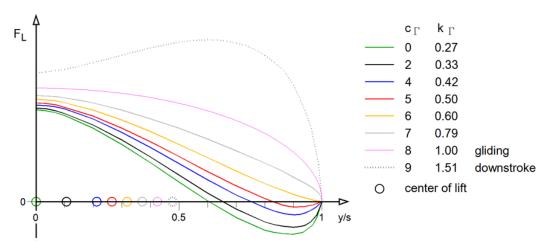


Figure 8. Various lift distributions for the upstroke in the wind turbine mode For comparison also are shown the following lift distributions:

 $c_{\Gamma} = 0$, with balanced torque

 c_Γ = 8, as an example for the gliding flight

 $c_{\Gamma} = 9$, as an example for the downstroke

The circulation factor k_{Γ} (k- Gamma) describes the size of lift based on that of gliding flight.

In this comparison the lift distribution with the distribution parameter^A respectively circulation characteristic number $c_{\Gamma}=5$ has the lowest induced drag. The length of its propeller area approximately equals to the free length of the primary feathers in large birds. This lift distribution also equates to those which was delineated by Otto Lilienthal (see following Figure 9). However, this has not been proven so far with technical measurement neither on birds nor on technical flapping wings.

^A Please see handbook "Wie Ornithopter fliegen" equation 2.4 and 2.6

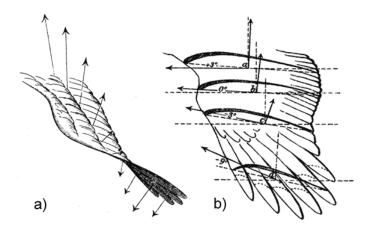


Figure 9. Two examples of lift distributions on wing upstroke by Otto Lilienthal⁵ (1889)

In drawing "a" however, the forces near the wing root are directed too much forward.

If on upstroke the torque of the wind turbine area is not complete compensated by the opposed torque of the propeller area, the force balance must be performed somehow in another way. Otherwise, without counterforce the lift force cannot develop on the wing. One must look then for other applications of the excess wind turbine energy (see chapter 10).

3. Time sinusoidal motion sequence

In the above Figure 8 only lift distributions of the upstroke are shown how they are in the middle of the stroke motion. In which way the transition between up- and downstroke can be archived, is not determined with it. But in general, the ornithopter theory assumes a temporally sinusoidal curve of the motions and the aerodynamic conditions. In the following Figure 10, the flapping motion of the wing is shown in the form of its stroke or angular velocity ω together with the respective stroke angle Φ . The change of the lift distribution takes place, at least with aeroelastic wing twisting, depending on the angular velocity. In its course are specified sample values of circulation characteristic numbers c_{Γ} at different times and below are shown the relevant distributions of lift in small format.

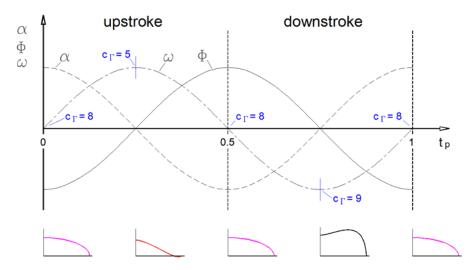


Figure 10. Basic, temporally sinusoidal course of the flapping wing motion

- Φ stroke angle
- ω angular velocity
- α angular acceleration

One must, however, be aware that under these conditions the mentioned lift distributions of up- and downstroke (see Figure 8) are valid only for a very short moment in the middle of the stroke. In the remaining time, so for about 99% of the flapping period, takes place a lift displacement between these forms of distribution - a slightly unusual thought. But from aerodynamic point of view, the always displacement of lift is the essential of the flapping flight. The specification of a "lift distribution for the upstroke" is actually misleading because it only applies for an instant of time. But there it will be needed for the description of the upstroke.

Informative to this is the analysis of the flight of a Dun Crow in the following Figure 11, by Hans Oehme⁶. The almost equal distances between the wing tip positions on downstroke show that the stroke velocity is not temporal sinusoidal, but nearly constant.

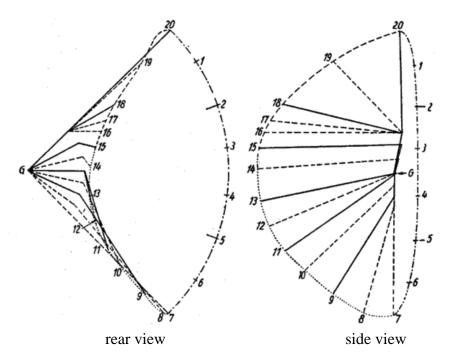


Figure 11. Diagram of the wing motion of a Dun Crow, with the shoulder joint "G" and the trajectory of the wing tip, from a movie in slow motion by Hans Oheme⁶. It is probably a flight with a high demand of thrust.

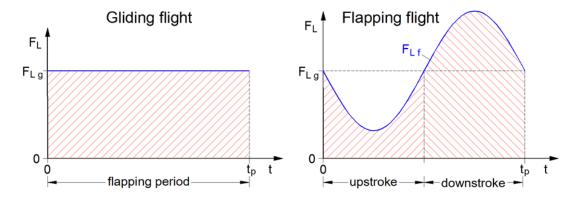
A largely constant rate of downstroke velocity has a major advantage over the sinusoidal course. The strong thrust and big lift exist not only for a short moment in the middle of the stroke, but remain for a longer time. In addition, with the same average lift, the maximum lift and thus also the requirements for the wing profile are smaller (see also Figure 13).

4. Lift impulse

To assess the effect of a force during the flapping flight, one must take into account not only its size but also the duration of its effect. For this purpose, is formed the product of force multiplied by time. The result is called force impulse.

force [N] x duration of action [s] = force impulse [Ns]

In the following Figure 12, the lift impulse during gliding flight in the distance of one stroke period t_p is shown on the left. On the right side you can see the lift course in principle with the same total lift impulse as it results from the commonly used theory of bird flight (see Figure 4).



The areas under the courses of lift in glide and flapping flight in the distance Figure 12. of a flapping period t_p corresponds to the relevant lift impulse. In flight practice both lift impulses must be from the same size.

In the following Figure 13, in addition to the sinusoidal course of lift of the two stroke cycles (blue), also is shown the respective average value^B (red). The whole lift impulse (hatched) of the two strokes cycles corresponds to that of the gliding flight.

If the stroke velocity is kept constant over a longer distance of time, here for example over the half of cycle time (black), the maximum value of lift can be reduced, in this case by ten percent. The size of impulse thereby is kept. For the upstroke is to choose a suitable lift distribution, which then remains constant over a longer period of time.

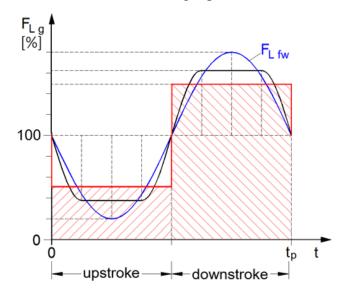


Figure 13. Course of lift $F_{L \text{ fw}}$ on a flapping wing, which is driven by the motor during upstroke. The scaling of the lift force is related to the lift of gliding flight, with $F_{Lg} = 100\%$

^B average value of a sine half wave = its peak value \cdot 2 / π

The minimum lift in the middle of upstroke (20% of lift in gliding flight) used in the Figure 13, exists for ornithopters if the flapping wing is powered by the motor during upstroke. The wing is then forced to work in propeller mode (see for this Figure 7). In order to achieve nevertheless a lift impulse in the size of gliding lift during a flapping period, the average lift of downstroke must be drastically increased (here to 149%). The maximum value in the middle of the downstroke is even at 180%. In addition, the maximum local lift coefficient along the span is already approximately 20% greater in downstroke than in gliding flight (see Figure 8). In sum, the maximum local lift coefficient on the downstroke would have to be twice as high as that in gliding flight.

Therefore, for the lift it is not advisable to drive the wing with a motor during upstroke. Instead, it should flap up with a lot more lift by itself. For this purpose, the wing is to operate in turbine mode and to brake its upward motion. Latter can be done by loading the motor with the tensioning of a compensation spring during wing upstroke (see chapter 10) or by keeping the rotational speed constant with a speed governor (see chapter 11).

The extremely high demands on the lift generation during upstroke are still increased by another effect. Both wing halves are during the flapping motion only for a short time in approximately extended position. But in the rest of the time, the lift force on the wing is not only directed upwards, but also slightly inwards or outwards (see Figure 14).

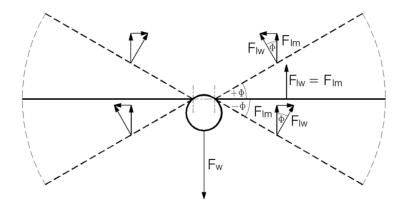


Figure 14. Position of the lift force outside the extended wing position

 $\begin{aligned} F_{lw} & & \text{wing related lift force} \\ F_{lm} & & \text{model related lift force} \\ F_{w} & & \text{weight force of the model} \end{aligned}$

As a result, the model-related lift F_{AM} becomes smaller. In a weakened form also the inclinations of the lift force towards the front or rear reduces the lift effect. With a flapping angle of \pm 30 degrees and sinusoidal course of motion, the model-related lift effect overall is about

8%^C smaller than with the extended wing. This lift reduction must be counterbalanced by greater lift generation and is an unavoidable disadvantage of the flapping wing technique. Therefore, it is advantageous that the lift on upstroke during the displacement outward increases automatically (see Figure 8).

The maximum lift of downstroke in Figure 13 cannot be reached with the permissible lift coefficients of the common profiles alone. As a rule, the chord of wing must be about doubled in comparison with a normally loaded wing in gliding flight. Also, for strength reasons therewith the wing weight increases accordingly. In gliding flight then you can fly only with low wing loading and hence only with low speed, at last with optimal angle of attack. The profile drag is doubled. For the thrust balance is necessary a higher flapping frequency. During the upstroke you must, so to speak, haul along the strong enlarged wing area, although the lift generation thereby is significantly smaller than in the gliding flight. Moreover, because of large lift on downstroke the torque of the gear increases, too. Also, the current of motor and the electronic controller are affected of it. In addition, the model shows an ugly up and down oscillation. An alternative is a drastic reduction of the flight weight (see Table 1). So, on upstroke generate only a very little lift has a whole series of considerably disadvantages.

Another option to boost the lift is to increase the flight velocity. But that increases the profile drag and this continues to require a high driving performance on downstroke. In this case there is also affected the parasitic drag from the enhancement of the drags. At the same time the change between gliding and flapping flight becomes more difficult.

While maintaining the lift distributions and the duration of the flapping period lift and thrust also changed on modifying the cycle time ratio of upstroke to downstroke. In shortening the downstroke, the total thrust of up and down stroke get smaller and the total lift increased (see handbook⁷, chapter 8.5 and Figure 9.6). As will be described in the following however, the decreasing thrust already caused enough problems in the improvement of the efficiency. Therefore, the variation of cycle time ratio is not further analyzed here. With high thrust models however, it is a possibility to increase the lift. The strength of thrust is also affected

^C The value was determined with a small addition in the calculation program "Orni 1".

by the flapping frequency. But also, that is not the issue here. It is generally assumed here from a mean constant value^D of the flapping frequency.

5. Changing the size of lift only with wing twisting

In calculation program "Orni 1" on a rectangular wing only twistings are calculated which arises during the flapping motion in flight by the change of freely selectable lift distributions. Other wing motions are not included. Thereby with each displacement of the lift automatically also change its size. With the help of the circulation factor and its connection of shifting and resizing, the angle of incidence at the wing root is kept constant. Therefore, one works only with wing twisting.

Thus, by keeping the angle of attack on the wing root the lift becomes always smaller in its displacement in the direction towards the wing root. Hence, thereby one should not displace the lift too far on upstroke to the wing root. However, then the additional drag is still relatively large. The changes become clear in Figure 15.

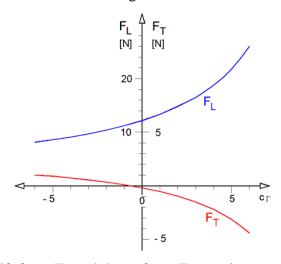


Figure 15. Course of lift force F_L and thrust force F_T on wing upstroke as a function of the circulation characteristic number c_{Γ} (c-Gamma) in an example of a rectangular wing. Calculated with the computer program "Orni 1"⁴

$$f = e^{\log \frac{10}{m_b}}$$

with the mass of the bird m_b [kg], see

Hertel Heinrich. Structure-form-movement. New York, Reinhold, 1966

The greater the displacement of the lift, the greater the thrust change with variation of the flapping period.

^D The mean value of the flapping frequency f_m [Hz] of birds is about

In the increasing concentration of lift in mid-span one can easily imagine that the flow along the wing always is getting stronger. In the calculation program however, this will be largely balanced by the ever decreasing lift. The induced drag increases so only slightly. If one however, increased the lift on upstroke, the cross-flow or the induced drag becomes a problem. Birds apply as a countermeasure the bending of the hand wing. This then acts as a winglet or end-plate and so reduces the flow along the wing and the wing tip vortex. To reduce the flow along the wing on ornithopters already may be helpful instead of a bending even a wing fence between arm and hand wing.

In attempt to rise the lift on upstroke I see the limit of the lift displacement to the wing root about in the circulation characteristic number $c_{\Gamma}=5$. The lift size is then still 50% of the lift of an elliptical distribution (Figure 8). A further displacement towards wing root only by wing twisting does not make sense. This is true at least for a rectangular wing with homogeneous profile along the whole span. For ornithopters 50 % of gliding lift on upstroke is certainly a quite passable value. But in this case achievable thrust is usually sufficient only for level flight or a very flat climb flight (unless one is working with high flapping frequency).

When increase lift on upstroke, generally one must keep in mind also the lift on downstroke. It then can be smaller. Beside adjustment of the lift distribution for this is also suitable a lower flight velocity. At least in the first case this means less thrust.

6. Rotation of the wing root

6.1 Size of lift with rotation of the wing root

By rotating the wing root one can further increase the lift generation by wing twisting on upstroke. In calculation program "Orni 1" thereto one must remove the connection between displacement and resizing of the lift. This can be achieved by entering of appropriate values in the actually not designated as an input parameter "circulation factor" ($k_{\Gamma 1} = 0.5$ to 1.2). The size of lift can selectively modify in this way. At the same time changes the angle of incidence at the wing root. But that only applies to the wing rotation with the maximum in the middle of the stroke. In slow motion shots of birds, I could not recognize such a rotation so far in the past. Erich v. Holst however has suggested it to equalize the lift.

In this way one can increase the lift on upstroke, e.g. so far, till the lift coefficient reaches the maximum value of a conventional profile on the wing root (see example with $c_{\Gamma} = 5$ in the

following Figure 16). While maintaining the balance E of forces in a level flight with a rotation of about +5 degrees (only on upstroke, maximum in the middle of the upstroke) one reached about 77% of the lift in gliding flight. Compared to the process only with wing twisting this is significantly more. With a strong chambered profile near the wing root and / or large wing depth in this range, the lift can be increased further more. Not for nothing birds in the arm wing section have a strongly cambered profile, maybe with a very high profile lift gradient (d_{CL}/d_{α}) .

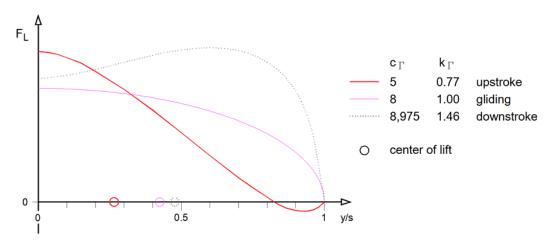


Figure 16. Rough approximation to a constant lift in the flapping flight, by an upstroke with $c_{\Gamma} = 5$ and an increase of the wing root angle by +5 degrees.

Therefore, by the combination of wing twisting and rotation of the wing root one can increase the lift on upstroke. But this only works with a moderate demand of thrust or with relatively high additional drag.

An increasing of the angle of attack at the wing root acts in wings with low torsional stiffness particularly in close range, so in the arm wing. Outside, the angle of attack gives somewhat way to the rising lift. But a rotation of the wing root always should be considered in the construction of the wing. The wing twisting on upstroke then can be accordingly smaller.

ornithopter.de

^E Changed values for the balance of forces in the calculation program "Orni 1": upstroke circulation characteristic number $c_{\Gamma^1} = 5.0$; upstroke circulation characteristic factor $k_{\Gamma^1} = 0.772$; downstroke circulation characteristic number $c_{\Gamma^2} = 8.975$; flight speed factor $k_V = 0.980$

6.2 Lift in the stroke end positions

As a distinctive intermediate stage between up and downstroke, particularly for calculations, one can still specified a middle lift distribution, applicable for both strokes end positions. Therefore, is suitable the lift distribution of gliding flight. This seems to be plausible, because at least a straight flapping wing of an ornithopter comes to a standstill between the two stroke directions for a short-time. At the same time, the gliding situation is a good guideline for assessing the changes on the flapping wing in the diagrams. But for the end position of ornithopters the gliding flight situation is not always correctly described with an elliptical lift distribution. This is only a first approximation. Also, a distribution with e.g. $c_{\Gamma} = 7$ (maybe as well gliding seabirds) brings very good results in gliding flight.

The wing twisting of birds by the anatomy is fixed in the glide position in the extended wing position (see chapter 8). If with the downstroke motion varies the direction of incoming flow, however, the wing twisting gives way in an elastic manner. Also, with muscle strength are still possible small changes in the twist. But in the upper final stroke position in the short moment of the wing standstill, when the hand wing has reached the extended wing position, as a rule, the lift distribution of gliding is probably also present in birds.

In the lower end position, the situation in birds is not so quite clear. The pivoting and bending motions of the hand wing has usually already set in there. But one can assume that shortly before reaching the lower stroke end position, the wings are still extended and the downstroke motion is already very small. Also, in this case is then approximately given the glide situation. Thus, also in birds one can approximately describe the wing end position with the glide condition.

For the transition between the two stroke cycles it is interesting that on large birds in cruise flight sometimes can be seen a slight pendulum motion of the angle of attack by the bird's body. Its lift generated together with the tail thereby is varied. Because the wing roots on both sides of the body follow this pendulum motion, also synchronously is changing their angle of attack. Approximately, the minimum angle of attack is at the upper and the maximum at the lower stroke end position.

The following Figure 17 was made from a slow-motion picture of a Greylag Goose. The red image shows the bird in the upper and the blue image in the lower stroke end position. Both images were aligned with each other to the eye of the Greylag Goose. As one can see, head

and tail keep on virtually the same relative level, while the neck and the chest move clearly up and down. Consequently, the body's angle of attack is changing.

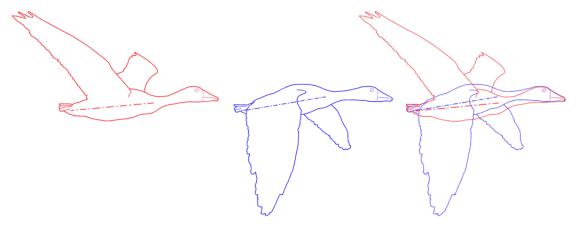


Figure 17. Nodding or swinging of the body of a Greylag Goose with illustration of the body axes.

Drawing based on a slow motion shot by Lloyd Buck⁸

When searching for changes in the angle of attack at the wing root therefore one should not look for motions of the wing's trailing edge compared to the bird's body. There virtually don't take place a relative motion. Also, the position of the leading edge of the wing relative to the bird's body does not change. One must try to determine the centreline of the bird body. But videos exactly from the front would be the best. On those one can direct compare the height of the leading and trailing edge of the wing (see Figure 19 and Figure 25 and the animations of flying birds based on films by A. Piskorsch⁹). To estimate the centre line of the bird body is only a makeshift.

The increased angle of attack at the beginning of the upstroke leads to a larger lift at the wing root. This is an indication that birds have already shifted for the upstroke a substantial part of lift of the downstroke to the wing root at this time. At this point, birds often even fold together the hand wing. Conversely, the decreasing of lift on the wing root in the range of the upper stroke end position allows us to conclude, that in mid-span a substantial part of the upstroke lift has been reduced and displaced toward the wing tip, at least till to the arm wing. In both cases, the corresponding displacements of lift along the wing thereby must take place by local modifications of the angle of attack.

The nodding motion of the bird body is generated by forward and rearward displacement of the centre of lift compared to the centre of gravity. This occurs partly automatically. As is known, on cambered profiles the centre of pressure with large angle of attack moves forward and with reduces angle of attack toward the rear. In addition are playing a role the wing sweep or the pivoting of the hand wings backward (see chapter 8), the inclination of the stroke plane (see chapter 9) and the moment of inertia of the fuselage. Beside also play a part the forward or rearward directed forces near the wing tip in the range of the stroke end positions (see handbook⁷ "How Ornithopters Fly", Figure A 14, in German). Therefore, body oscillation is a very complex process.

The nodding motion comes about to a standstill in the stroke end positions. Accordingly, the nodding speed is greatest about in the middle of the stroke. In this way as a result of the nodding motion downward the incoming flow angle will be enlarged at the leading edge of the wing during the upstroke. Thus, the lift is somewhat kept high even still in this time domain. On downstroke that reversed. Both in sense of lift displacement are assessed as positive.

The axis of rotation of the nodding motion of the goose in Figure 17 is close to the trailing edge of the wing. Alternatively, to increase the angle of attack also the axis of rotation could be moved further forward. However, a tendency to flow around the trailing edge in the direction to the upper side of the wing possibly may be occur (see Figure 18). This then led to a small, short-time separation of the flow on the wing's upper surface.



Figure 18. Possible tendency to flow around the trailing edge of the profile with rapid increase in the angle of attack around a far forward positioned axis of rotation.

From the aerodynamics, however, I don't know such a flow behavior. There, the lift changes almost instantaneously with the change in the angle of attack (with an unspecified position of the axis of rotation). Maybe the unknown is in the word "almost". In any case, the example from biology shows the axis of rotation of the goose wing at its trailing edge and this is certainly no coincidence.

This arrangement of the axis of rotation means at each flap an up and down motion of the body mass. Its lifting is associated with work. But the bird can convert altitude by gliding into thrust. It may be that the lift on upstroke cannot make big enough, because just now a lot of thrust is needed. The missing lift then must be compensated necessarily by larger lift

at the downstroke. The resulting up and down motion of the body mass is used in this case to achieve even a small aerodynamic advantage.

6.3 Wing motions of a swan

Something different looks the rotation of the wing root if one analyzed the images of a swan from a movie clip by A. Piskorsch⁹ (see following Figure 19 and Figure 25). Unfortunately, the image material is slightly blurred and the result corresponding inexactly.



Figure 19. Swan from the front, based on a movie clip by A. Piskorsch⁹

In Figure 20 the end of the upstroke was set on the beginning of the downstroke motion of the arm wing (course of stroke angle Φ). The upstroke motion of the hand wing has practically ended there (course of bending δ). In this figure shows increase and decrease of the angle of attack a very well the temporal sequence of lift displacement between wing and midspan.

The maximum angle of attack in this case occurs not, as in the goose in the lower stroke position, but in the first half of the upstroke. It is been built up together with the bending of the hand wing in the early stage of the upstroke motion. The increase already starts towards the end of the downstroke. A distinct minimum as at the goose in Figure 17 cannot be seen in the upper stroke position. In this wing position be more existent an angle of attack as it might be present also in gliding flight. It is there almost constant for a longer time. Changes of the angle of attack at the wing root on an elastic twistable wing affects especially in close range, thus on the arm wing. But, if one look also at this low, even though over a longer time relatively constant angle of attack as a minimum, the two mentioned cases of goose and swan are not so different.

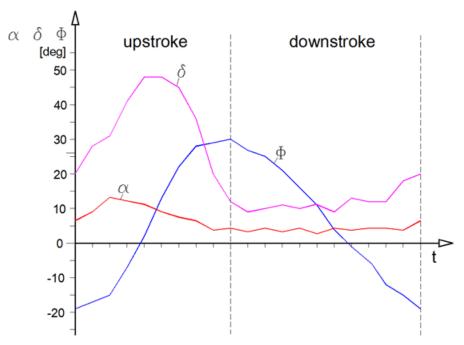


Figure 20. Wing positions during a flapping period of a swan in cruise flight based on a movie clip by A. Piskorsch⁹

 Φ stroke angle of the arm wing compared to the horizontal

 δ bending of the hand wing compared to the arm wing

 α angle of attack^F on the wing root or bird's body

The cycle time ratio indicates that the bird is doing everything possible to generate enough lift.

In Figure 20 the angle of attack is especially heightened during the upward motion of the arm wing at the wing root. During this time, the lift is mainly generated only in the arm wing, thus in a partial length of the wing. Therefore, the rotation of the wing root also can be considered as method to keep the lift constant. However, the increased lift works in wind turbine mode. But because of the strong concentration of the lift near the body or of the small lever arm of the force, the performance is low. Thus, the additional drag remains small despite of large lift.

The bending δ of the hand wing compared to the arm wing does not go back to zero on downstroke. This is due to the type of image analysis. The wing of the swan is curved downward also on downstroke along the whole half span. Arm and hand wing have been replaced each by straight lines. In this way the bending of about 10 degrees remains on downstroke.

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^F Strictly speaking, you have to add a few degrees to this angle of attack, since the footage was taken from a bridge, so from above.

One should actually move the whole course of the bending in the diagram by 10 degrees downward. The maximum angle of bending is then only 40 degrees. Presumably, the swan is in an acceleration phase, maybe after a start. In unaccelerated cruising flight the bending is only about half as large.

It is remarkable that the bending of the hand wing already started far before the downstroke of the arm wing has ended. The reason for this is partly the decrease of the lift forces in the outside wing area. The hand wing thereby springs elastically a few degrees downward. In addition, it can also not be fully excluded that the bending of the hand wing, at least at the beginning is done by muscle power. But due to the weak muscles to move the hand wing (K. Herzog¹²) this is very unlikely.

After its upstroke motion the arm wing takes a break above. It is a kind of transitional phase. Only the hand wing is still moving upwards. At the end, in the extended hand wing position the wing twist is mechanically fixed (see chapter 8). Without downstroke motion of the whole wing it is then like in gliding flight. In this way in the transition phase with the relatively powerless upstroke motion of the hand wing is displaced the lift to the outer wing area and so prepared for the downstroke.

6.4 Phase shift of the lift displacement on the wing root

Considering only the stroke angle Φ and the size of the lift in form of the angle of attack α , one has the impression that both variables do not behave according to the familiar Figure 10. They seem to run nearly independently of one another (see following Figure 21). But the offset times of distinctive basic parameters of these distributions pointed all in the same direction. The start of the enlargement of lift runs ahead of the start of the upstroke motion of the arm wing by the time span φ^*). The maximum of the lift is by the time φ still before the middle of the upstroke motion. Also, the end of the lift enlargement is brought forward by about the same time φ^{**}). Therefore, one can speak on upstroke of a phase shift φ between stroke motion and the lift displacement on the wing root.

At the end of the upstroke motion of the arm wing it takes a waiting time ($\approx \varphi$). In this phase of the transition to the extended wing position (δ) lift is shifted from the arm wing in the hand wing. At the same time is finished the lift shifting from the mid-span (α). Subsequently, the lift shifting within the wing can run again depending on the flapping motion during the downstroke.

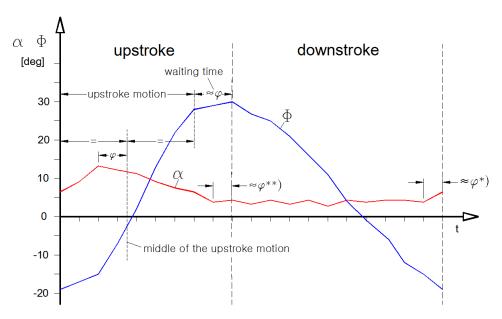


Figure 21. Phase shift φ between the lift displacement at the wing root and the stroke angle of the arm wing

- Φ stroke angle of the arm wing related to the horizontal
- α angle of attack on the wing root or of bird's body
- *) by φ earlier beginning of lift displacement
- **) by φ earlier ending of lift displacement

The buildup of lift in mid-span can be described rather concretely with reference to the rotation of the wing root. At the same time, however, according to the previously usual theory, a reduction of the lift in the outer wing area should take place. Instead one can see in the individual images of the swan that the wing bending starts towards the end of the downstroke, even during the primary feathers are slightly curved upwards (see Figure 25). In the lower stroke end position therefore is existent appreciable lift at the wing root and in the outer wing area at the same time. This is still a bit unusual. In addition, then possibly is required the application of muscular work.

When assessing the phase shift, it should be taken into account that in the based diagram (Figure 20) were simply used the instants of times of the several film images. In addition, there is only this one slightly blurred image material for the detection of the phase shift.

It looks as if the phase shift of the lift displacement is practiced by the Swan to achieve a big lift in the lower stroke end position. The lift is indeed built up early in the mid-span, but only delay reduced in the outside located wing area. Thus, the lift can be concentrated in the mid-span when the hand wing is still not bended. The speed of the upstroke motion thereby is still

low. In the range of the upper stroke end position the lift is mainly displaced during the waiting time of the arm wing in the outside wing area, so virtually with no upstroke motion of the arm wing. In this both time segments of the upstroke therefore are generated considerable lift with very little additional drag.

The decrease of lift in the outer wing area in the lower stroke end position takes a relatively long time. The upstroke begins, still while the primary feathers are slightly bent upwards. Nevertheless, it is possible that at the beginning of upstroke, at the time of the maximum lift in the mid-span (Figure 21), much of the lift has already been displaced along the wing in the direction of the wing root. Thereby then there is already negative or at least very little lift in the outer bended wing area at this time. In this way the bending of the hand wing is supported by the change in its aerodynamic forces at an early stage.

The shift of lift in the range of the stroke end positions simply needs its time.

6.5 Compensation of the inertial force of the wing

One possible reason for the above-mentioned phase shift of lift displacement is the compensation of the inertial force of the wing. It results, because the change in the stroke or angular velocity of the wing mass. During the braking of the mass the inertial force acts in, and when accelerating against the direction of motion. The maximum inertial force is directly in the end position (see the course of acceleration α_B in Figure 10 and Figure 22).

One possibility for reducing the inertial force is the articulated connection of arm and hand wing. As a result of the short partial lengths of the arm and hand wing, their radius of gyration and their masses are correspondingly smaller. Both together have a very strong influence on the moment of inertia of both wing parts. During the acceleration process of the arm wing, indeed you cannot completely neglect the mass of the hand wing, hanging on it. Nevertheless, it remains a significant reduction of the moment of inertia of the flapping wing. The two wing sections also do not reach the upper end position at the same time, but only successively.

In the range of the upper end position, the inertial force at the end of the wing upstroke acts as an increase of lift. The aerodynamic lift may or may not be adapted accordingly. The additional lift does not damage. But the bending moment because inertia should be taken into account in the material strength of the wing.

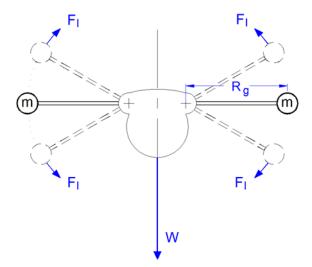


Figure 22. Concentration of the mass "m" of a wing half on the radius of gyration R_g and its inertial force F_I in the end positions. The radius of gyration of the flapping wing is the distance from the axis where its mass is concentrated while maintaining the moment of inertia.

Contrariwise, the force of inertia on the radius of gyration acts as an additional weight in the range of the lower end position. This is only true, however, if the wing mass is braked by the drive mechanism, thus from inside of the ornithopter (e.g. with an end position spring). If, on the other hand, braking takes place from the outside, thus by the aerodynamic lift on the wing, then is generated no additional weight. It is therefore very advantageous to compensate the inertial forces in the range of the lower stroke end position by lift. However, this must be sufficiently large and should be accordingly verified.

The percentage of the maximum inertial force of the Ornithopter in the "Orni $1^{"4}$ computer program is \pm 70% related to the lift in gliding flight. The lift in the size of the gliding lift in the end position, therefore, is sufficient for a sinusoidal motion sequence in order to break the wing in the lower end position without "weight increase". The inertial force is relatively large in this case. But it has been calculated with a constant weight distribution along the whole wing. Many flapping wing structures will have a smaller distance to the center of

gravity. However, it is not easy to determine the moment of inertia which is required for the calculation. A proposal for practice is made in the Handbuch⁷, chapter 5.6^G.

In birds, the moment of inertia is clearly smaller than in ornithopters. The reason for this is the small distance of the gravity centre of the wing from the shoulder joint and the smaller wing weight. Nevertheless, it may come to noticeable inertial forces. In the wing-downstroke of the Dun Crow in Figure 17, the deceleration or negative acceleration near the lower end position only takes place between the positions 6 and 7. However, when the braking time and braking distance are shortened, the inertial force increases. To illustrate the increase, the description of the acceleration work for the wing mass is helpful. At the same initial conditions, the acceleration work for reaching a certain velocity is always the same.

$$\mathbf{W} = \mathbf{F} \cdot \mathbf{s}$$

W work of acceleration [Nm]

F force of acceleration for overcoming the inertial force [N]

s distance of acceleration [m]

Thus, as the distance of acceleration becomes smaller, the force of acceleration increases. If, for example, the angle or the distance for braking the wing mass is reduced to one-fifth, the inertial force increases to five-fold, at least with constant acceleration. Nevertheless, it is unlikely that, in this example, the lift of gliding flight in the stroke end position of birds is not sufficient for breaking of the wing. In the case of ornithopters with shortened braking distance of the flapping wing, however, a check is generally recommended.

The effect of the inertial force can also be played on yourself. To do this, you place yourself on a bathroom scale, which should have an analogue display, if possible, and flaps up and downwards with your arms. On the display of the scale then you can read the described changes of your own weight.

6.6 Rotation of the wing root on ornithopters

For ornithopters it will not be easy to copy the nodding motion of the bird's body. But with a long lever arm of the tail it is probably anyway better to work first with a rotation of the

The same error was also corrected in the computer program "Orni 1", version 4.0 and also supplemented the calculation of the inertial force.

^G Warning! The equation 5.12 in the manual it should read $J_F = \frac{m_F}{3} \cdot \left(\frac{b}{2}\right)^2$

wing root relative to the fuselage. But in which period the angle of attack at the wing root should be increased. At least in the case of straight flapping wings it is not necessary to initiate a bending of the hand wing (see chapter 7.2).

The swan data in Figure 20 as well as the illustration of the Greylag Goose in Figure 17 are only single cases of not closely described flight situations at any one bird species. So, it is still unsure what temporal progression of the angle of attack at the wing root under which conditions is more suitable. As long as there are no measurements in a wind tunnel, one must probably approach to the optimum by means of experiments. However, the course of the angle of attack α in Figure 21 is certainly a good point of reference for its size and the temporal progression. The influence of the change in angle of attack on the wing root to the size of the wing twist during upstroke has to be considered in the construction of the flapping wing. The wing twisting thereby can be smaller.

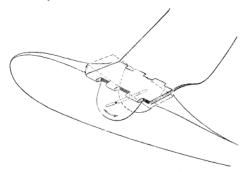


Figure 23. Suggestion for construction of a wing root rotation by Karl Herzog¹⁰, from his series of articles "Der Schwingenflug in der Natur und in der Technik", Nov. 1963

In the ornithopters EV1 to EV5¹¹ I also used a rotation of the wing root (about \pm 3 degrees). The maximum has been in the middle of the upstroke and the minimum in the middle of the downstroke. The temporal course was sinusoidal. According to today's knowledge it was wrong to use such a rotation of the wing root during downstroke or at least it was too big.

7. Bending of the hand wing downward

7.1 Bending in general

In cruise flight of birds, the bending of the hand wing downward indeed remains relatively small. But also, in this case it certainly offers some advantages. They are described in more detail in the following.

By the end plate function of the hand wing the lift in the range of the arm wing is kept together. Thus, the bending helps on the concentration of the lift in mid-span and therewith on the generation of thrust. But this only being worthwhile if there is actually high lift. Without strong lift in the area of the arm wing and without strong lift differences along the half span a strong bending is less reasonable.

As a result of the bending changes the direction of the lift force on the hand wing. Thereby the forces on both wing sides of the bird balanced partially each other. The effective lift in the vertical direction thereby decreases (see Figure 24). With the most common negative lift this is advantageous, with positive lift disadvantageous.



Figure 24. Direction of the lift forces on the hand wing when its bending

blue with positive lift red with negative lift

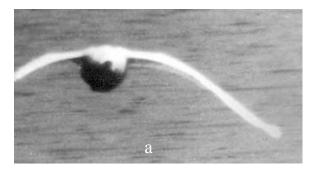
With the bending of the hand wing, its distance of the centre of gravity from the pivot of the wing becomes smaller. This also applies to its centre of lift. Together with the just described reduction of the lift effect, this also affects to the wind turbine function of the flapping wing (this does not always have to be advantageous for braking of the upstroke motion). Its influence decreases. This also applies to the inertial force of the wing in the upper end position (see chapter 6.5). This reduces the need for the application of energy storage devices (chapter 10) when bending the hand wing.

The conception of flapping wings can be optimized if one let move self-actuating the many motions of the single wing parts. The bending of the hand wing mentioned here can be coupled very favorably mechanically with the twisting of the arm wing (the larger the bend, the larger the twist of the arm wing). Birds certainly, partially work in that way too. The bending in turn can be driven by aerodynamic forces. In the ornithopters¹¹ EV6 to EV8 for this was used the shift of lift between the arm and hand wing (bend only \pm 3 degrees). But for a larger thrust generation, the bending on upstroke should be made significantly larger.

To design mechanically a strong wing bend for a level flight of ornithopters seems to be rather disadvantageous. But even with a small wing bending the appearance of slowly flapping ornithopters looks beautiful. Then it looks somewhat like the great role models.

7.2 Bending during the upstroke

One method for setting in motion the bending of the hand wing is shown in the following Figure 25 of a swan. It shows the wing in the lower end position on the left side and the wing quite a while after the beginning of the arm wing upstroke on the right side (on the second instant of time on the timeline in Figure 20). The angle of attack in the figure on the right is much larger than in the figure on the left. The tips of the feathers in Figure 25b are still slightly bent upwards.



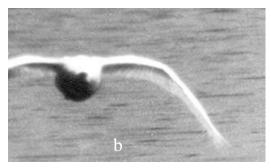


Figure 25. Beginning of the bending of the hand wing by increased lift in the arm wing, from a series of pictures by A. Piskorsch⁹

Therefore, only a strong lift must exist in the arm wing area. A considerably smaller lift in the area of the hand wing does barely impair this method. On the contrary, the hand wing tends to rotated by the upward motion of the arm wing around its centre of gravity. Thereby the wing tip will be flapped inwards and generates even a little thrust near the tip. An involvement of muscle strength in the process is rather unlikely. In birds, the requirement to initiate the wing bending is, in addition to the lift generation, certainly an important reason for the early enlargement of the lift on the wing root.

On upstroke with spreads wings are valid the lift distributions of Figure 8 and according chapter 6.1 in particular this with $c_{\Gamma} = 5$. Which lift distributions are advisable on bendable hand wings is unknown. However, one will be endeavor to maintain the bend initiated in the lower stroke end position throughout the whole upstroke. As long as no tests and

measurements in a wind tunnel are possible, one must approach by the method of trial and error to suitable angles of attack.

With sufficiently negative lift, a bendable hand wing turnout downwards from the incoming flow from above. The lift thereby will be not as negative as in an extended flapping wing. Nevertheless, it is advisable to limit the bending. The size of bending of the hand wing in birds depends on the requirement of thrust or the respective flapping frequency.

With ornithopters, one can achieve a similarly flexible limitation of the bending when the axis of the wrist is inclined slightly inward at the rear (see following Figure 26). Thereby grows up the angle of attack of the hand wing during the bending and the resulting increasing lift reduces this motion. If for example the hand wing is bended by 90 degrees downwards its angle of attack is increased by the angle λ . In this way, the hand wing comes to a standstill in an intermediate position. With the size of the additional angle λ therefore can influenced the size of the bending. It also accelerates the upward motion of the hand wing in the upper stroke end position. Approximately 10 degrees for the additional angle may be a useful starting basis for experiments.

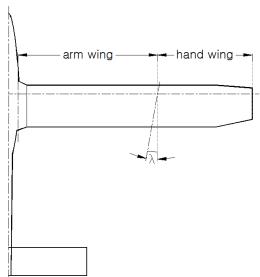


Figure 26. Additional angle λ (Lambda) of the wrist of an ornithopter in order to achieve an increase of the angle of attack in the hand wing during the bending.

A construction with a very large additional angle shows following Figure 27. It replaces the wing twist in the forearm of the birds by a variable profile kink along the torsion axis (see also Figure 29). During the upstroke with maximum profile kink, this construction simultaneously provides the maximum bend of the wing. One can well imagine that in this state

much lift is accumulated at the outer end of the forearm. In the upper end position then this lift assists by pressure compensation the upstroke of the adjacent hand wing. In this way the lift will be displaced in the direction of the wing tip.

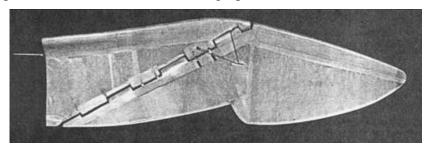


Figure 27. Flapping wing with modification in camber of profile and the angle of attack, in particular on the forearm, on bending of the hand wing. Two rubber threads on the bottom side of the wing supports the bending.

Construction by Karl Herzog¹⁰ 1963

Sometimes it looks, as if birds support the bending of the hand wing by muscle power, at least at the beginning of the motion. In ornithopters this can simulate by a spring which bends the hand wing towards the arm wing a little way downward. K. Herzog has done this with two rubber threads (see Figure 27). The strength of the spring one can choose so that it just even can be full extended by the lift in gliding flight. Possible effects on landing have to be considered. But a better method for the bending shows Figure 25.

7.3 Wing spreading in the upper stroke end position

On upstroke with a bending of the hand wing it is clear, that the arm wing reached the upper stroke end position before the hand wing. In slow motion videos of large birds then can be seen that the arm wing is waiting at the top until the hand wing has nearly reached the extended wing position (see e.g. Figure 21).

The waiting time of the arm wing in the upper stroke end position, with the included upward motion of the hand wing and the lift displacement, is part of the transition from the up to the downstroke. Without this delay, the bending may include a very high mechanical load.

In Figure 28 on the lift side is shown the motion sequence of the hand wing in birds. In that the hand wing moves upwards, as shown in position 1. The arm wing goes ahead of the hand wing and reaches in the position 2 the upper stroke end position. Subsequently, the arm wing remains in this position and is waiting until also the hand wing has reached the upper end position 3. In this time by changes of the angle of attack as a result of the hand wing motion

lift will be shifted in the outside wing area. With the extended wing position then starts the downstroke with full thrust generation.

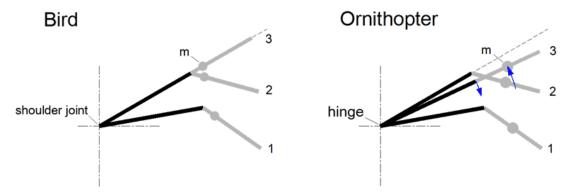


Figure 28. Comparison of the motion sequence of the hand wing in the range of the upper stroke end position.

In current ornithopter suggestions the drive generally is not stopped on reaching of the upper stroke end position. Thus, the arm wing does not wait until the hand wing is at the top. In addition, in contrast to the bird's wing the centre of gravity of the hand wing is located not so close to the wrist. This results in the following scenario, shown on the right side of Figure 28.

The hand wing moves upward as shown in position 1. The arm wing is going ahead of the hand wing and reaches in position 2 the upper stroke end position. The drive immediately drives the arm wing downwards. But the hand wing is still in upward motion. It reached the extended wing position in the position 3 and strikes hard there with its opposite motion against the end stop of its pivot. At the same time the stopped mass of the hand wing is a very sudden obstacle to the downstroke motion of the whole wing. The far outside located hand wing first must be accelerated by the drive to the previous downstroke speed. To all this be added an abruptly increasing lift on the hand wing. Correspondingly large is the impact load on the wing spar and the gearbox. Only then starts the full generation of the thrust. The bending should increase the thrust. But if the first part of the downstroke barely generated thrust, the goal is not reached.

There are several ways to alleviate the problem of inertia of the bended hand wing:

a) Let the arm wing wait at the top until also the hand wing is reached above, like in birds.

For this purpose perhaps uncouple the wing from the drive or stop the drive for a

short time. One also can try it with a drive which under every increasing of the crank torque first always tensioned a spring, before the crank turns further.

- b) On crank gears use drives with a speed controller. Because of the crank characteristic the power requirement on the motor in the range of the crank dead center is low. Beside this, the hand wing is not loaded there. The drive should not react with a speed increase in the range of the crank dead center. So, the slowed down motion combined with sinusoidal course should replace the waiting time here.
- c) Make the mass of the hand wing small and concentrate it as close as possible to the wrist.
- d) Use a soft mechanical end stop in the wrist.
 Also elastic wing spars can help. This however causes undesirable stroke oscillations.

8. Pivoting of the hand wing to the rear

Besides the bending of the hand wing downwards one sees in birds also a pivoting motion of the hand wing to the rear. In general, both bendings are applied simultaneously.

In birds, the twisting or even rotation of the hand wing is severely restricted on the wrist by the anatomy¹² in the extended wing position (by wing skeleton, tendons, edge ligament for inclusion of the primary quills). Without downstroke motion then there is a distribution of the angle of attack along the whole wing like in gliding. However, an elastic twisting of the hand wing, for example on downstroke is still possible and also a little additional twisting by muscle power. The restriction of the twist will be stronger the further the hand wing tip is pulled forward by the thrust. Only by an at least small pivoting motion of the hand wing to the rear will be loosened this restriction. Its big advantage is the fast and nearly powerless setting of the wing twist for the downstroke and the gliding flight. In addition, in birds together with the pivoting motion to the rear occurs simultaneously an area reduction and taper ratio by superimposed pushing of the primary feathers.

With the pivoting motion of the hand wing rearward the length of the arm wing is automatically shortened slightly (see following Figure 29). This changes the wing profile at the elbow. Probably, the position of the maximum profile thickness, the chamber of profile and the angle of attack are changed. Also, the sweep of the forearm displaces lift in the direction of the wing root. Under these circumstances, these changes can support, or in case of lower

requirements, even replace, the displacement of lift by rotation of the wing root. Overall, by the change in shape of the wing the centre of lift will be shifted more towards the wing root. This is good for the concentration of lift in mid-span, but for ornithopters hardly to apply into practice.

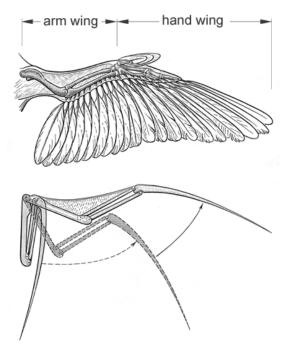


Figure 29. Pivoting motion of the bird's wing, drawn by Karl Herzog

Also, the wing as a whole performs a small pivoting motion related to the body. The drag on upstroke pushes it rearward. The thrust on downstroke, especially in the outer wing area, pulled it forward. The pivoting motions of the hand wing and this of the whole wing run synchronously and add up. So, a trajectory of the wing tip like in Figure 30 feigned a too large bending of the hand wing, in particular forward.

With the pivoting motions of the wing also the centre of lift moves backward and forward. Thus, the bird is raised at the backside during the wing upstroke. At the end of upstroke then the bird's body and thus the wing roots are inclined slightly downward (see Figure 17). During downstroke with its wing pivoting and displacement of lift forward the bird's body is then erected again.

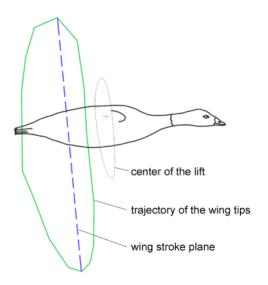


Figure 30.

Brent Goose in cruise flight with the trajectories of the wing tips and of the centet of lift.

The centre of lift was assumed here at 25% of the chord at the wing root.

As long as one does not use the nodding of the fuselage on ornithopters, this restricts a little the benefits of the pivoting motion of the hand wing. But the involvement in the end plates effect remains. This is probably true, especially if is executed a taper ratio at the end of the hand wing. As however, a strong sweep or an oblique inflow of the hand wing effects on its negative and positive lift is unknown.

Birds also use the pivoting motion for curve controlling. Thereby they influence in particular the twisting elasticity of the hand wing. After attempts by E. v. Holst¹, the turning works in flapping flight alone by greater or lesser elasticity of the hand wing on one side of the wing. With greater elasticity on the desired curve outside, there results a greater wing twisting and the stroke motion gets easier on this side. The flap angle gets bigger here and automatically smaller on the other side of the wing. In this way, on the outside of the curve the thrust increases and the curves begin. By stronger incoming flow on the outside of the curve, the lift increases there and the wing side will be raised. Thus, prevents a skidding turn (slip to the outside). So, it comes to different ranges of stroke on the right and left side, but not by different stroke intensity or different mechanical stroke motion. Instead of the elasticity of the hand wing, at a pinch, one can only change the maximum of its twisting for the curve control of a model.

In contrast, according to E. v. Holst, birds for curve control in gliding flight often use reducing of the wing area on the inside of the curve. This is also possible only with a pivoting motion of the hand wing to the rear (with one-sided displacement of the lift center to the rear). Due to the increasing elasticity of the hand wing at the same time its angle of attack

becomes smaller. Thus, it is not clear which of these changes is decisive for control. For ornithopters, it is easier at first only influence the size of the twist.

In this way, with the same sweep motion of the wing the curve control happens in the opposite directions in flapping and gliding flight. This needs getting used to the radio control pilot. Furthermore, however, the living bird still has a great wealth of other options for position and course control. By Konrad Lorenz comes the sentence, that the bird always achieves the same effects of control in a different way.

The position control about the transverse axis or the pitch control is possible by forward and backward displacement of the wing area. The main problem of the position control is to keep the balance. This is solvable with computers, but difficult to do. Thus, a stabilizing elevator will remain essential for a while.

9. Inclination of the wing stroke plane

The wing stroke plane is an imaginary plane which is sweep over by the wing axes during the flapping motion. An inclination of the wing stroke plane can be achieved in two ways. Either the inclination of the flapping axis to the axis of the fuselage is fixed installed or one turns the fuselage axis in relation to the direction of flight when flying. In the latter case, the inclination of the wing stroke plane is quasi created by flying. The difference between the two methods lies in the behaviour of the angle of incidence along the whole wing. If the inclination of the stroke plane is created by flying it changed, but not with the installed inclination. In the following only the installed inclination of the wing stroke plane is considered and short labelled as "inclination".

In cruise flight of birds not always can be seen the inclination of the stroke plane. If so, it then runs mostly from rear-top to front-bottom. As shown below, this has advantages for the generation of thrust. But the wings are moved not only in the stroke plane. Especially at the outside they are also pulled forward by the thrust on downstroke and pressed back by the wing drag on upstroke. The result is an approximately elliptical trajectory of the wing tip, whereby the longitudinal axis of the ellipse lies in the stroke plane (see Figure 31).

A great benefit of the inclination is the increase of the approach velocity on downstroke in the outside located wing area. The small forward motion of the wing tips is added to the flight velocity. Because the lift increases quadratically with the speed, the inclination is absolutely significant. It helps on downstroke to increase the lift and thus also the thrust force in the outside located wing area. A particular advantage of this is that thereto must not be extended the working area of the lift coefficient of the profile. During upstroke the smaller approach velocity in the outside located wing area supports the downsizing of the lift.

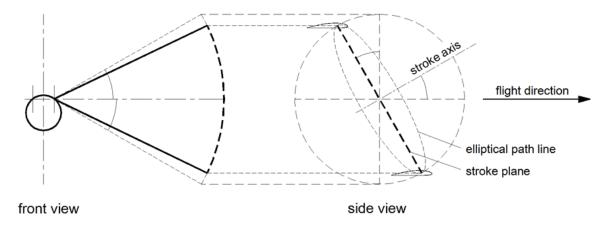


Figure 31. Installed inclination of the wing stroke plane by tilting the stroke axis in relation to the fuselage axis.

The inclination therefore helps in the lift shifting along the span and thus in thrust generation. With increasing inclination, however, decreases more and more the up and down motion of the wing (see Figure 31). This comes at the expense of the thrust generation. As showed in the calculation of a large ornithopter model, a weakly pronounced climb flight optimum is at an inclination of about 10 degree. But at the same time there is a minimum for the distance of flight (see handbook⁷ "Wie Ornithopter fliegen", chapter 8.8, Fig. 8.13).

10. Energy storage with springs

A first reference on saving the upstroke energy in case of technical aircrafts comes by Otto Lilienthal⁵. In his suggestions for the construction of flying apparatus he wrote among other things (in German):

"30th.— It would be of advantage to store the effect of air pressure during the upstroke so that it may be utilized again during the downbeat, and thus save work."

Thus, when a spring is arranged so that is tensioned by the wing on upstroke and it remove the tension with the downstroke it fulfils this requirement (see following Figure 32). The cycle of wind turbine energy then can be described as follows.

On wing upstroke in the wind turbine mode, thereby occurring additional drag reduces the airspeed of the flight model. Thus, detracts the kinetic energy of the model mass. Via the upstroke motion of the wing the relevant amount of energy is stored as tension energy in the

spring. During the downstroke the spring tension is removed and gives the energy back again to the wing. There it is converted into thrust. The thrust accelerates the model and gives back the kinetic energy to the model mass. The energy related to the additional drag during upstroke therefore is not lost. It can be recovered with the aid of elastic elements.

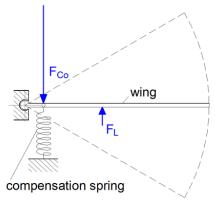


Figure 32. Arrangement of a pull spring as lift compensation spring F_{Co} = force of the compensation spring F_L = lift in gliding flight

This energy cycle of course is lossy. However, the losses of the wing upstroke are also existent if the wing worked in the propeller mode. The induced drag by the calculation program in the propeller mode is even throughout greater than in the wind turbine mode. The advantage of the whole story with the wind turbine mode is the significant lift that can be generated also on upstroke.

Because the centre of lift of the wind turbine force lies always near the wing root, the thereby converted energy in the spring is generally not very large. Therefore, the spring can be relatively small. It just needs to balance with its average force the average torque of the in wind turbine mode operating wing. The spring is then able to absorb all generated energy of the wing during the upstroke. Only a small problem thereby is the unequal forces of wing and spring during the upstroke motion. The drive therefore must take over the force balance and the controlling of the upstroke velocity throughout the whole upstroke. But on average it should idle during the upstroke. With a real speed governor this may be possible.

If indeed one already used such a spring, it can also be used very advantageous for the storage of drive energy. To achieve that, the spring must be of larger dimensions. If it is then tensioned not only by the wind turbine function of the wing but also by the drive during the upstroke, it also stores that energy. The spring supports the drive on the downstroke and releases thereby the stored energy again.

The drive is loaded by the spring during upstroke and pushed during downstroke. The wing forces, or more precisely their torques, worked just inverse to the drive. In this way the drive system is operating more equally and with a considerably smaller peak load during a flapping period. Thus, motor and gear can be dimensioned significantly lighter. With a wing upstroke in wind turbine mode, thus can approximately halved the peak load of the drive system in flapping flight.

The spring will be advantageously dimensioned in a way that it just balanced the torque of the average lift during a flapping period in the middle position of the wing. This average lift during a flapping period equates about to the lift in gliding flight. Furthermore, the spring force should be relatively evenly during the whole flapping period. Therefore, the spring rate should be chosen as small as possible. However, a steel spring is then relatively strong, large and heavy. A gas spring, despite of its worse efficiency, may be better in this case.

The spring compensates the lift which exists in average on the flapping wing. To distinguish it from other springs in a flapping wing drive I call it lift-compensation spring, or just as "compensation spring".

The compensation spring also facilitates the fixing of the wing in glide position during flight. However, when starting and while decreasing the lift force during landing it pushes the wing tips down. If for holding of the extended wing position the cogging torque of the standing motor is not sufficient as a brake, so there is necessary an additional brake or lock mechanism.

In each flap between the two end positions first the wing mass must be slowed down and then accelerated again in the opposite direction. In the technique in such cases are used springs. One obtained so an oscillating system that keeps moving in the theoretical ideal case (without damping by the wing area and without friction) without external supply of energy. The drive is then no longer loaded by acceleration forces.

To be able to hide mentally easier the influence of gravity in the oscillation in a corresponding experimental arrangement, the swinging mass of the wing is shown suspended vertically in the following Figure 33. Thus, weight is neglected.

On the flapping wing acceleration forces are superimposed by the lift forces. In the lower end position, the lift supports the deceleration and acceleration of the wing mass. In the upper end position, it works in opposite to the motion reversal.

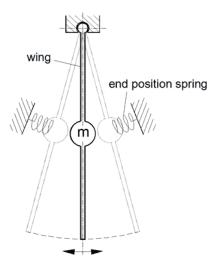


Figure 33.
Swinging wing mass "m"
between two end position springs

In practice, one should take the following way. The spring of the lower end position will be simply omitted (see Figure 34). The deceleration of the wing mass at the end of the downstroke can be taken by its lift (see chapter 7.2). Subsequent the lift of upstroke drives the wing mass in the opposite direction. In this way, also the "additional weight" of the acceleration is avoided (see chapter 6.5).

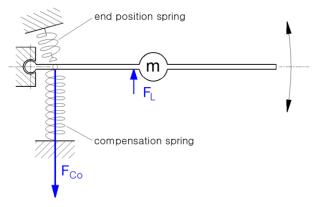


Figure 34. Arrangement of compensation spring and end position spring on a flapping wing

The compensation spring here is designed as a pull spring and the end position spring as a pressure spring.

 F_{Co} = force of the compensation spring

 F_L = lift in gliding flight

The upper end position spring first is sized for their task to accelerate the wing mass. For this purpose, the inertial force of the wing with its radius of gyration (see chapter 6.5) will be proportionally converted to the spring force and its lever arm. If no separate compensation spring is used, it is necessary to strengthen the end position spring so that it can also counteract the lift force. For this, the end position spring in the compressed state will be

additionally strengthened dimensioned by the force which is necessary to balance the lift on the wing in gliding flight. The lift in the short-term standstill of the wing during the motion reversal corresponds theoretically about to that of gliding flight. So, the end position spring can perform the acceleration of the wing mass and simultaneously act towards the interfering lift force.

The kinetic energy of the wing mass will be absorbed by the upper end position spring during decelerating. Subsequently, when the wing mass accelerates in the direction of downstroke, the spring releases its tension energy back to the wing. In this way it supports the drive on downstroke and also with the thereby generated thrust the acceleration of the model mass. So, the end position spring acts nearly like a shortened compensation spring. With a suitable spring characteristic both spring functions also maybe combined in only one spring. The use of an end position spring is worth especially with large wing weight and not bendable wings.

The wind turbine energy during the upstroke can be used by direct generation of thrust in the outer wing area as well as with the help of compensation springs or end position springs. All three methods also can be combined. Indeed, the mentioned springs increase the model weight, complicate the construction and make difficult the handling, for example, during a test run. Hence one should also look for other solutions.

In birds, some kind of springing could facilitate the idea, how it is possible for them to glide for hours with extended wings without much muscle power. So far, unfortunately nothing is known to me about it. On the other hand, in birds during flapping flight, the demand for energy storage by appropriate configuration of the upstroke may not be so great. Only in the range of the upper stroke end position also in biology a kind of end position spring will assist the transition from upstroke to downstroke.

11. Usage of a speed governor

In propeller driven model aircrafts, the drive controller works mostly as a normal speed controller. With it you can vary the rotation speed however the setting speed varies also with changing of the load. In contrast, real speed governors maintain the set speed constant even under changing loads.

A really rpm-controlled flapping wing drive provides several advantages. In the normal case the drive is virtually very unevenly loaded during a flapping period. Only the wing downstroke requires the full drive power. In the case of non-controlled drives therefore the speed increases significantly during upstroke. Moreover, the most commonly used cranks in the mechanism, in the top and bottom dead centre does nearly require no drive torque^H. There the motor runs practically at idle. It responded with a significant increase of speed of rotation. Additionally, are coming torque oscillations which results from elastic wing spars. The moment of inertia of the motor smoothed the speed curve slightly (With an outrunner motor or with a flywheel you can increase this effect and even use it for the storage of wind turbine energy.). But one certainly cannot speak of a continuous or even sinusoidal motion process during a flapping period when using unregulated drives. On flying ornithopters you even can hear the non-uniform drive operation.

If the lift during upstroke is large enough, also with a running motor the wing can be the propulsive part. It then tries permanently to accelerate the motor. This now works as a generator. From the preset target speed, the brake of the controller becomes active and keeps the speed constant. The thereby imparted wind turbine energy can be used, depending on the controller type, to convert it into heat or to lead it with an energy recovery system into the battery. In the latter case the excessive upstroke energy will be buffered in the storage battery. It then can be used again on downstroke.

However, the energy balance of regenerative brake worsens considerably by the effectiveness of the mechanism, motor, electronics and battery on the round trip of the energy (overall efficiency lower than 50%). In addition, the mostly used cranks derogated this method by their motion characteristic. Motor and gear will be also not unburdened during downstroke. The gear even must be strengthened, because it must withstand the changing load directions. Nevertheless, a regenerative brake can in principle take over the storage function of a compensation spring.

When it is possible to keep the excess wind turbine energy small, also a real speed controller is useable with a brake that converts the energy into heat, at least at the beginning the development. The controller, however, is to protect against overheating.

^H Already E. v. Holst¹ has determined that the motion and force progression of a simple crank is unfavorable for driving of flapping wings. He has compensated this with eccentrically mounted winding plates in his rubber powered crank drives (e.g. see his model "Bussard"¹¹).

12. Requirements for the ornithopter construction

A requirement for a significant lift generation during upstroke is a suitable wing design. Whether the wing can work in wind turbine mode depends crucially on the upstroke velocity and the thereby existing distribution of the angle of incidence along the span. The wing must be able to keep positive angles of attack in spite of existing lift.

This requirement is not so easy to fulfil with wings that change the distribution of the angle of incidence along the wing depending of the lift (aeroelastic wing). In this aeroelastic method the *wing* area twisted around a stiffness torsion axis, which is typically the spar. Thereby the lift force counteracts a torsional force of the wing. The size of twist is determined by the size and location of the lift force inside the chord of the wing, the elasticity of the used components and the location of torsion axis of the wing. If the torsion axis lies far forward an increasing lift, for example on downstroke, strengthened the torsional moment. At the same time the pressure point of the lift force moves forward and so reduces the torsional moment. So, the size of the lift force and the position of its pressure point behave contrary. Therefore, an aeroelastic twisting works only reliably if the torsion axis is positioned far ahead. In addition, it is not easy to achieve a specific twisting, because the lift force is changing along the wing and the mechanical properties of the wing structure against torsion mostly are missing. With an aeroelastic wing, however, lift can be generated absolutely also on upstroke. The wing just has to present already in unloaded condition a strong positive angle of incidence.

The aeroelastic twisting has the advantage that it can adapt flexibly, also without a sensor-system, to different directions of incoming flow or flight situations. In the most known flapping wing designs the hand wings are exclusively twisted aeroelastic. However, also the transition between up and downstroke should be made stepless. Only then can be fully utilized the adaptation qualities.

But you can also control the wing twisting, by the drive mechanism^I or by servos^J. Also, it can be used the displacement of lift^K along the half span. Each of these methods has advantages and disadvantages. But always comes due to the elasticity of the components, intentionally or unintentionally, an aeroelastic twisting component to it.

To minimize the required thrust, first of all you must try to make the parasite drag of the ornithopter as small as possible. In the field of fuselage and tail an aerodynamic design is still relatively easy to implement. But in the case of the wing this means almost inevitably the transition from a wing membrane to a profile with a good lift-to-drag ratio. For a high effectiveness of the flapping flight good profiles are even indispensable. They improve also the often-practiced gliding flight of the ornithopters, but restricted in flapping flight the possibilities for strong thrust in the outer wing area.

In the usual profiles the lift coefficient has only a relatively small operating range. Unlike membrane wings they are able only in a very limited extent to work with strong negative lift coefficients. In general, profiles work only well with positive lift coefficients.

However, thick profiles are able to manage passable with positive and negative lift coefficients. But they have a relatively high drag. Nevertheless, one should not ignore them completely. Furthermore, in the outer wing area with the profile selection is to ensure, that in addition to the projected operating range of the lift coefficient also reserves are available. In practice the flight situations often differ to the intended. Thus, it is advisable to work only with lift distributions whose negative part is small. This is the case only with circulation

^I Already E. v. Holst (1940) has effectuated with his rubber powered crank drive of his flapping wing models not only the flapping motion but also the wing twisting (please see http://www.ornithopter.de/english/her-zog.htm#crank). Also the wing twisting of the ornithopters Truefly (please see http://truefly.chez.com/) and EV1 to EV5 (please see http://www.ornithopter.de/english/picture1.htm) has been controlled by their drives.

J On the reproduction of the Quetzalcoatlus Northropi (please see https://www.ornithopter.de/eng-lish/wings.htm#maccready) by Paul MacCready and on the SmartBird (please see https://www.festo.com/group/de/cms/10238.htm) servos were used for the continuous adaptation of the wing twisting. On SmartBird only the twisting of the long hand wings was active controlled by servos.

K The displacement of lift along the half span has controlled the twisting of the ornithopters EV6 to EV8. This applies especially to the thereby developed aeroelastic controlled articulated flapping wing (please see http://www.ornithopter.de/english/articulated.htm). Thereby the shifting of the pressure point along the chord plays only a subordinated role.

characteristic numbers c_{Γ} with values larger than 4 (see Figure 8). This further more limits the thrust generation.

To solve the problem with the too small operating range of the lift coefficient or the strong alternating approaching flow directions one has already experimented with artificial primary feathers at the wing tip^L. They can react more flexibly with their angle of incidence to changing approaching flow directions than a continuous surface. For the application of slats (e.g. Alula), flaps and other lift aids regrettably are still missing suitable flapping wing constructions.

Altogether, you can also look critical the changeover to more lift generation during upstroke. Instead of the lift problem there is a thrust problem. Rises off ground or steep climb flights are only possible by changing the mode of operation of the flapping wing. In addition, the technical requirements are relatively high. They can be summarized as follows:

- It is very advisable to use profiles with a good lift-to-drag ratio. Especially in the outer wing area they should have a wide operating range of the lift coefficient and if possible, they also should be able to work with negative angles of attack.
 A large wing depth along the whole wing span helps to increase the reserves of the lift coefficient. But it is combined with a major profile drag.
- 2. There is required a wing design which can maintain a positive angle of attack also at existing lift during the upstroke.
- 3. The displacement of lift along the half span is done in particular by a suitable wing twisting. But with this alone achievable concentration of lift in the mid-span suffices only for moderate thrust and moderate lift on upstroke. An inclination of the wing stroke plane supports the displacement in both stroke cycles.
- 4. On upstroke, for the concentration of lift in mid-span or for boosting thrust comes into consideration a rotation of the wing root (only on upstroke). In addition, can be provided a bending of the hand wing downward. With strong concentration of lift then also high lift no longer interfered with the thrust generation. Thereby helps a great wing depth and a strong profile camber near the wing root.

^L For example in my ornithopter model EV7b, please see http://www.ornithopter.de/english/picture3.htm#ev7b

- 5. On upstroke, the wing shall only be powered by aerodynamic forces. In normal case, even is required a force against the upstroke motion. Preferably this is done by negative lift with generation of thrust in the outer wing area.
- 6. On upstroke, with insufficient lift concentration in mid-span the excess wind turbine energy is to pass into a device with energy saving (e.g. springs, battery, flywheel) which then supports the downstroke.
- 7. For the downstroke, also outside of the wing the chord should be large particularly where in birds is effective the Alula.
- 8. To increase the thrust, the angular velocity can be maintained approximately constant during long distances on downstroke.
- 9. The parasitic drag of the whole aircraft must be minimized.
- 10. For curve control with the wings it is advisable to influence the twisting, especially in the outer wing area.

If one like to imitate the excellent flight performances of birds or simply want to fly very energy-efficient, one has probably given more attention to the generating of lift during the upstroke.

Information about the program "Orni 1"

In the computer program "Orni 1"⁴ is used the system of equations recognized in aerodynamics by R. T. Jones¹³. All lift distributions shown here were calculated with it. The program only applies only to the simplest way of flying, thus the unaccelerated level flight and the gently inclined climb flight. Furthermore, it is restricted to rectangular, straight, upswept wings under quasi-stationary flow conditions. These frame conditions are generally applied here.

The system of equations by R. T. Jones has the feature that the resulting lift distributions, taking into account the position of their centre of lift, shown the minimum of induced drag. This is also advantageous for birds. So, they probably work with similar lift distributions in cruise flight.

With the calculation program "Orni 1" you can view all corresponding distributions of lift coefficient, downwash, angle of incidence and angle of attack for a rectangular wing.

However, some terms there are otherwise defined (lift = transverse force, vertical force = lift, thrust or additional drag = propulsion).

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