# Arrangements of wing tip vortices on flapping wings

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## Content

## 1 Vortex arrangements on wings with infinite span

To get an idea of the flows and vortices on a flapping wing, let's look at first how come off the vortex picture on a conventional wing. Lift is generated by the inflow of a wing, which has a suitable airfoil and a positive angle of attack. Thereby is developed a higher pressure on the bottom side and a lower pressure on the upper side of the blade. Both pressures are trying to equalize each other by flowing around the wing tip. Thus a strong tip vortex is developed at this point. The flow around the wing tip is part of a cross flow along the whole wing. On the lower surface the air flows to the outside and on the upper surface to the inside. Thereby, the flow coming from the front will be deflected along the wing chord in the respective directions.

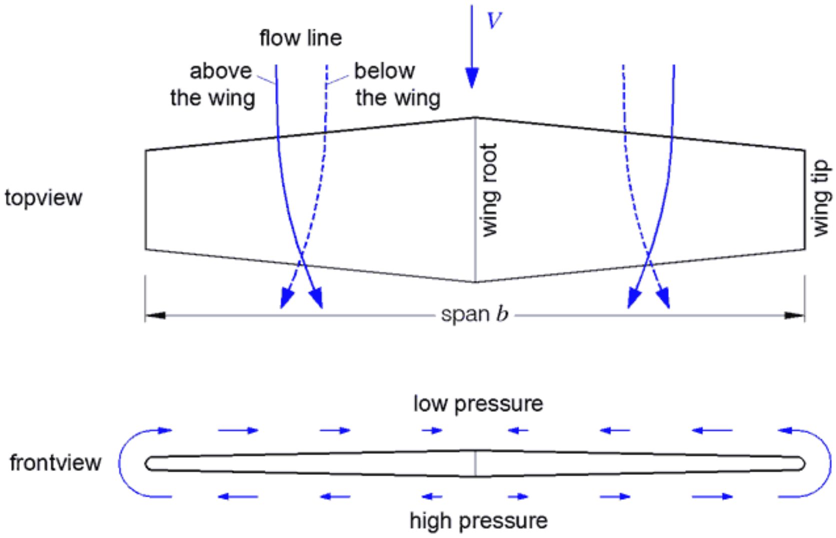

**Figure 1.** Flows on a wing.

In this image, are drawn some of the flow lines coming from the front. These air particles converge with the equivalent other directed air particles of the other surface side at the trailing edge. From then on, they will be twisted together. Also, one can imagine all these stream lines as threads which are twisted to vortex filaments at the wing trailing edge. Such vortex filaments evolve along the whole span. Together they formed a continuous vortex layer. This is pulled apart by the tip vortices like a rubber blanket and be coiled up in some distance behind the wing.

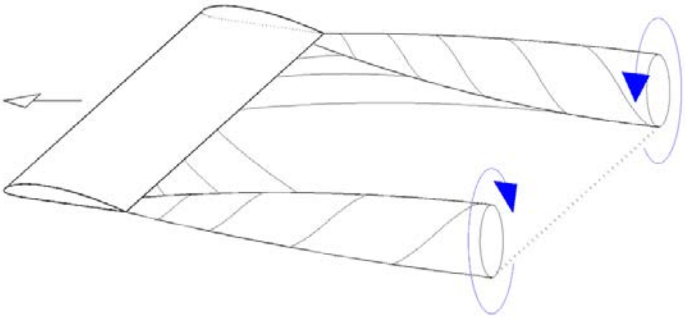

**Figure 2.** The vortex wake is coiled-up by the tip vortices behind the wing.

The wing tip vortices don’t generate lift. They are only a by-product of finite wings. The lift is already being developed on the wing. The pressure differences to the rest of the airspace above and below the wing transferred a downward directed velocity impulse on the form the wing included air mass. This induced downwash velocity is maintained behind the wing. It will be only overlaid by the flows of the wing tip vortices. Only the cause of the tip vortices, the cross flows, determines the induced drag and influenced the process on the wing.

The flow transverse to the approach flow depends on the respective lift distribution. The most famous is probably the elliptical shape of distribution. In wings with limited span it has the lowest induced drag. The following picture (Figure 3) of the vortex developed when one inscribes lift sections or stages of any but the same size in the lift distribution. Here there are three stages. The locations of the lift distribution corresponding to these values are plumbed down. They mark the position of single vortex filaments of equal strength along the span on the x-axis of the diagram.

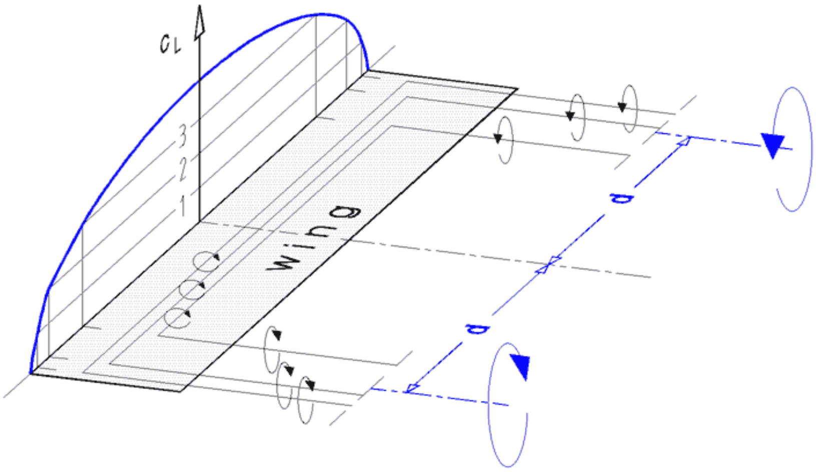

**Figure 3.** Determination of the tip vortex distance “d” from the wing center on an elliptic lift distribution. It is smaller than the half span.

They are called bounded vortex filaments inside the wing and free vortex filaments behind the wing. Every free vortex filament takes part of the movement that is induced by the other vortex filaments in that location. According to this vortex law the free vortex filaments with the same direction of rotation are twisted together in their common center at some distance behind the wing. So the average distance *d* from the wing root of the considered vortex filaments is the location of the resultant wing tip vortex.

This distance is only a theoretical value. It is only achieved when the tip vortex has rolled up all the vortex filaments along the half span. This occurs in some distance behind the wing. Furthermore, the internal friction and viscosity of the air is neglected. However, even so, one has a relative simple method to determine the approximate location of the tip vortex on straight wings. If the stages of lift are more numerous, the result will be more accurate.

It now becomes interesting when determining the distances of the tip vortices with lift distributions that are displaced compared to the elliptical distribution in the direction of the wing root or the wing tip. In the following Figure 4 are shown such lift distributions. For comparison, the elliptical distribution is drawn in the diagram.

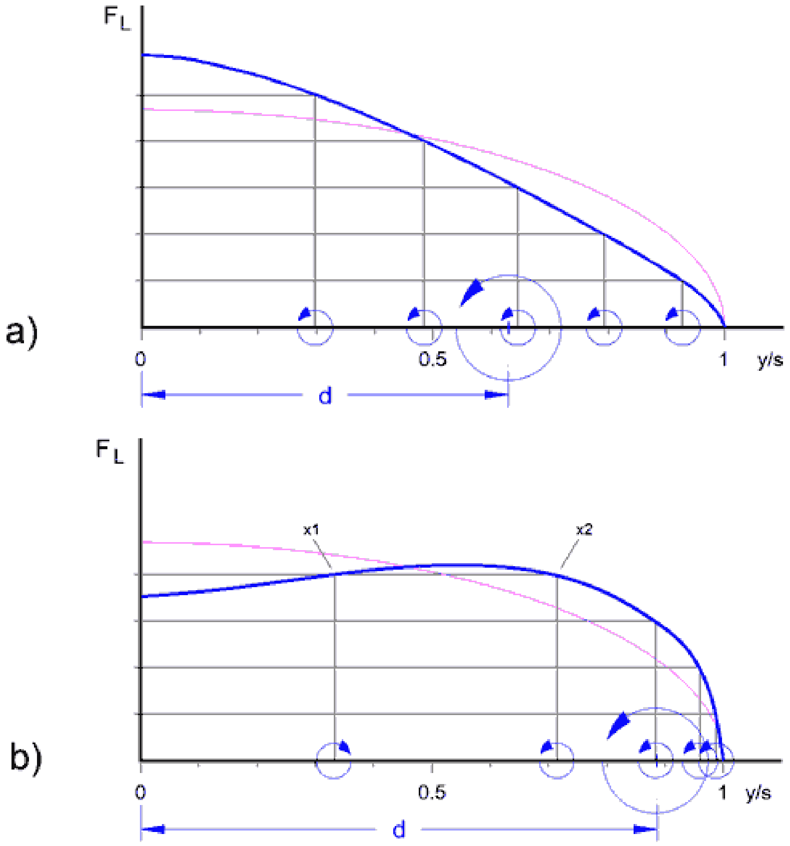

**Figure 4.** Positions of the tip vortex of lift distributions that are displaced compared to the elliptical distribution
a) shifted to the wing root
b) shifted to the wing tip
The total lift of each distribution curves is from the same size.
y/s = relative half span

One look at the wing root shifted distribution a) reveals that the “tip vortex" has clearly moved from the wing tip. In the wing tip shifted distribution b) in the highest lift stage there are two intercepts with the lift distribution. It has to be clarified how this affects the arrangement of the tip vortex.

When coming from the wing tip the lift increases quite sharply. There the vortex filaments are close together and they all have the same direction of rotation. Together they form a strong wing tip vortex. But the vortex filaments change the direction of rotation from the point where the lift distribution slopes inward towards the wing root. Therefore, the vortex filament at the inner intercept point x1 turns contrary to that at the intercept point x2. Furthermore, the vortex filament at x1 makes its own small double vortex with the corresponding vortex filament on the opposite wing side. With it does not develop upwash but downwash.

The small double vortex does not take part in the creation of the wing tip vortex. It only superposes it. Therefore, the small double vortex remains unconsidered in the calculation of the tip vortex distance. The two wakes shall influence each other further back in the wake of the vortex system. Certainly the small double vortex will be simply absorbed into the primary double vortex.

# 2 Vortex arrangements on birds

First, I would like to say that here I am only talking about large birds. In general are assumed quasi-steady flow conditions. Furthermore, I only considered here the simplest way of flying, the unaccelerated level flight.

## 2.1 Displacement of vortices by changing of the wing shape

There are relatively many studies about the wing movements during a stroke period. Regrettably, however, mostly only of smaller birds. They can easier be observed in the wind tunnel. Here are 4 detailed analysis results of slow motion films of a hooded crow in an acceleration period (see following Figure 5). Together with the knowledge of wings of finite span one can already well imagine the movement of the tip vortex.

About the wing downstroke in birds there is a broad agreement. The lift in this cycle should be relatively large and displaced towards the wing tip. So, the tip vortex at downstroke is certainly started close to the wing tip. The bird tries to keep it as small as possible by spreading the pinion feathers.

In the upstroke, it is generally assumed that mainly lift exists near to the fuselage. Thereby the wing more or less will be folded together. Here, in phase 15 the wing is folded together so far, that certainly there is broad lift along the whole remaining span. Consequently, like wings with small aspect ratio there is a strong flow around the outer edge of the wing. Thereby the back and down angled hand wing acts something like an end-plate or a winglet. In this way will be reduced the tip vortex and the corresponding induced drag.  Therefore also in the upstroke the location of the tip vortex agrees about with the position of the wing tip. Also the arm wing is widely shortened in phase 15.

Between these extremes of a stretched and a fully folded wing the tip vortex may sometimes move a quite a distance from the tip of the pinion feathers toward the wrist joint of the wing during a stroke period. But this makes only a little difference on its trajectory of a whole flapping period. It will then be only slightly different from the trajectory of the wing tips as presented here.

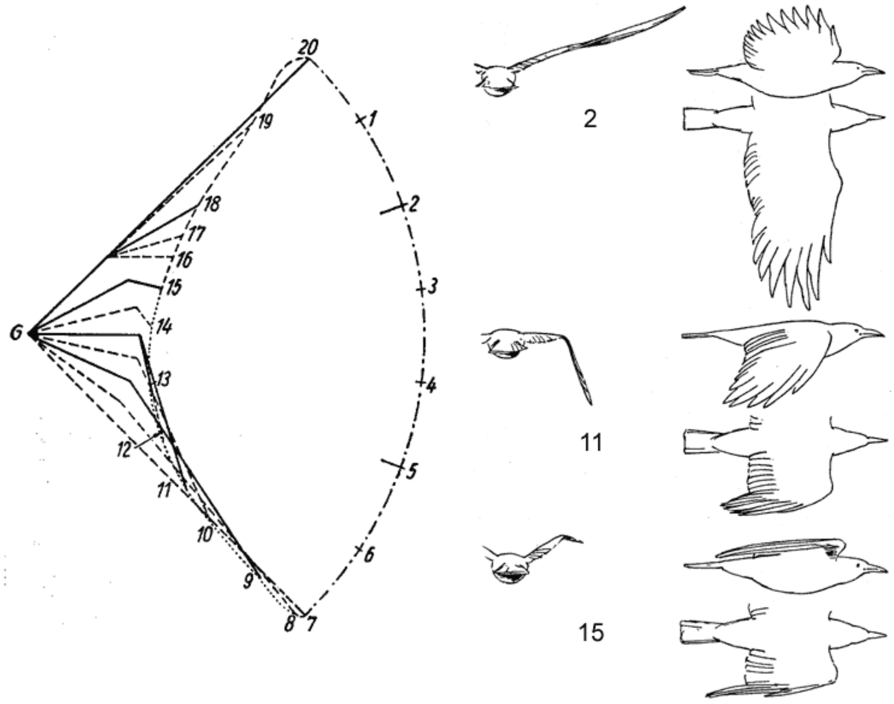

**Figure 5.** Images of the wing motion of a hooded crow during a stroke period. Starting from the shoulder joint G there are drawn the movement phases of arm- and hand wing. In particular, thereby will be visible the trajectory of the wing tip. In three here selected stages in each case the bird is shown in three views (images by Hans Oehme[1]).

Similar motion processes occur with many smaller birds. But the trajectories of the wing tips in the middle of the stroke do not always lie so far apart when beating up and down. Particularly at low thrust requirements, the wings are less widely folded together on upstroke.

## 2.2 Supposed lift distribution of birds at the upstroke

Searching for lift distributions of birds is difficult, in particular of large birds which fold up their wings only a little on upstroke. Measurements of pressure distributions are not possible so far on the wings of a bird in flight. For the wing downstroke there are text descriptions and even drawings[2, 3] (for the maximum thrust as in Figure 4b). But for the upstroke to my knowledge the only sketch of a lift distribution comes by Otto Lilienthal[4] (1889). He has imagined it with the wing of a stork. Currently, the usual text descriptions, however, are quite vague.

Mostly on upstroke lift is only assumed in the area close to the fuselage. Thereby, the outer wing section shall be raised with the feathered angle. There should be either no lift or this at least be widely reduced. This description indeed corresponds to the basic principle of the thrust and lift generation in flapping flight, but how one has to imagine the related technical relationships without detailed information?

First, I interpret here the statements about the specified wing sections so, that with the inner wing area is meant the arm wing and with the outer wing area the hand wing. In a stork the length of the hand wing is about 40 % of the half span.

In birds, the geometric angle of incidence seems to be relatively constant at the wing root, at least in the middle of the stroke. According to my own observations of slow motion shots it rarely changes between the up and the downstroke. Also at the transition to gliding, it appears that thereby this angle will change only slightly. Unfortunately, there are no detailed studies about the angle of incidence at the wing root. But in my estimation the above described lift distribution of the birds begins with a relatively high value at the wing root (almost as in gliding flight). Then probably the whole described lift distribution looks like in the following Figure 6a.

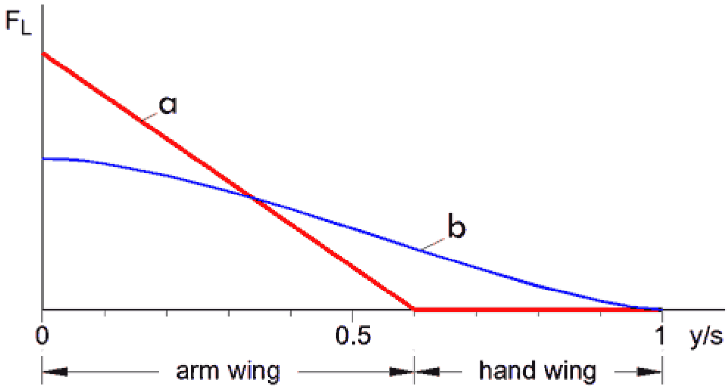

**Figure 6.** a) Approximated curve of Lift force $F_L$ as it shall exist according to the common bird flight descriptions.
b) An appropriate course in consideration of the cross flow along the wing.

On a wing of finite span cross flows are inevitable. At all events, Aircraft designers would give a great deal if they could avoid the induced drag coming with it. So when, as in the present case, the lift on the wing root is large, lift must exist along the whole span. At negative angles of attack also negative lift is possible in parts of the outboard wing area.

But cross flow also occurs when different strengths of lift lie close together along the span. In the present case a) this is particularly true. Hence, the cross flows will be very large. The cross-flows decreased if one approximated the distribution of the angle of attack to the lift curve b).

Lift in the outer wing area in upstroke means a relatively large additional drag. Obviously it is especially for this reason supposed as not existing in birds. In any event, mostly cannot be recognized in the descriptions whether the alleged lift distribution is based on measurements, observations or other bases. However, the additional drag only seemingly means additional losses. One can win back its related energy for the thrust generation. Even Otto Lilienthal has described one of the relevant technical possibilities, Erich von Holst[5] has mentioned in passing another. But that's another topic[10].

The additional resistance occurred at existing lift even in the arm wing section. With increasing distance of the considered wing location to the wing root, the angle $\delta$ is getting bigger between flight direction and blowing direction.  One can easily imagine in Figure 7, the increasing additional drag $F_T$ on upstroke. It is probably the best, if one takes both possibilities into account and, depending on the flight situation selects the most appropriate.

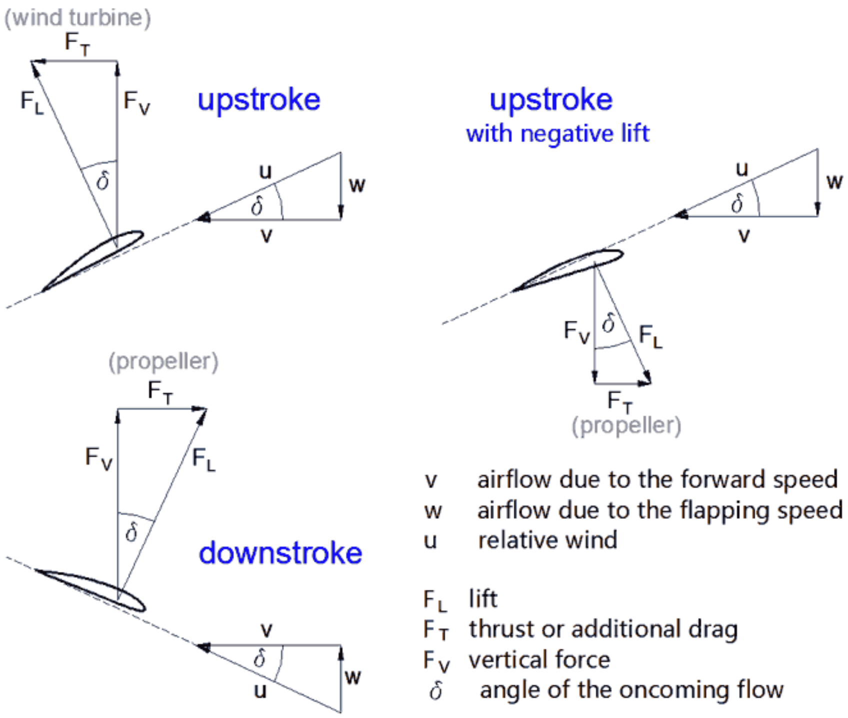

**Figure 7.** Forces at the flapping wing on a location in the outboard area. This is an illustration without airfoil and induced drag.

Another problem of the upstroke is that it must produce considerable lift. Otherwise, the bird's body would swing much more up and down than is noticeable during the long flapping period of large birds. In cruise flight of birds there is scarcely anything to see. In addition, the entire lift almost must be generated alone in the downstroke. But this would be possible only with an enlargement of the wing area. The discussion whether the wing during upstroke is lifted actively with muscle strength or passively by air forces has already been around since the beginning of the bird flight research.

A lift distribution as in Figure 6a is also supposed in ornithopters with membrane wings. Also in their case it is not possible, at least not on wings with an attached flow. Otherwise, during upstroke the membrane in the outer wing area would flutter as a sail which lies in wind direction. But about this nowhere is reported. However, for ornithopters there is the possibility to clarify this circumstance via wind tunnel measurements.

## 2.3 Displacement of vortices by changing the lift distribution

From what has been said so far you can come up with lots of lift distributions that may occur during the bird flight. The lift distributions which are shown here were created in applying the system of equations by R. T. Jones[6]. It’s also the basis for the small computer program “Orni 1”[7]. All lift distributions shown here have been calculated with this. With this program you can view all corresponding distributions of lift coefficient, downwash, angle of incidence and angle of attack for a rectangular wing. However, some terms there are otherwise defined (lift = transverse force, vertical force = lift, thrust or additional drag = propulsion).

The system of equations by R. T. Jones has the feature that all created lift distributions have linear downwash distributions. In this way, the deformation of the wake is small behind the wing. One can assume that the occurring induced drags will always be relatively small compared with other similar forms of lift distribution.

The here chosen distribution for the downstroke by R. T. Jones[8] (see following Figure 8) is optimal for the generation of thrust with respect to the induced drag at a given moment of force at the wing root. In comparison with an equal sized elliptical distribution, it supplies approximately 10 % more thrust at the same level of efficiency.

In the next Figure 8 a lift distribution was chosen for the upstroke, which has negative lift in the range of the pinion feathers. This helps to shift the lift more towards the wing root.  For the rest, this distribution is somewhat similar to the distribution shape of Figure 6b.

It’s important for the distribution shapes chosen here, that the geometric angle of incidence is kept always constant at the wing root. Under this condition, in the calculation method by R. T. Jones the change in size of the total lift results automatically with the displacement of lift[9].

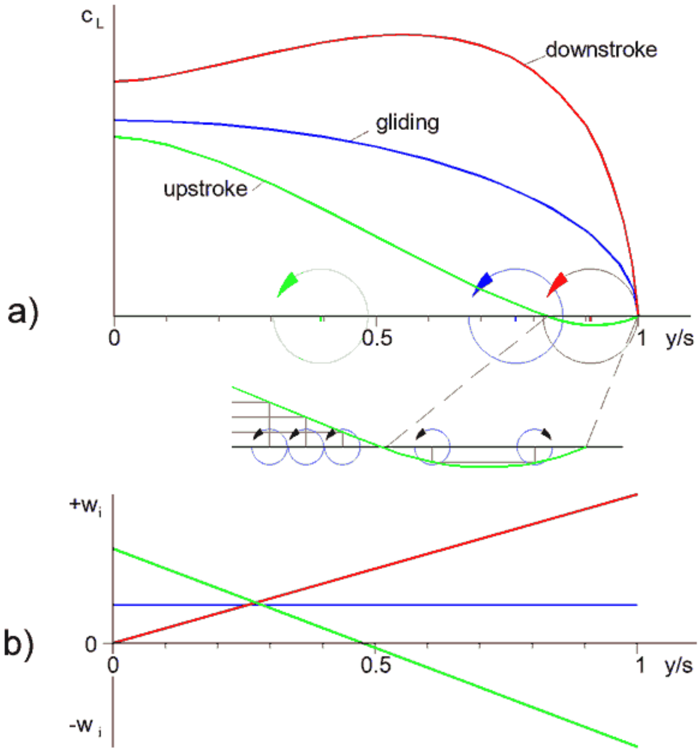

**Figure 8.** a) Possible lift distributions for flapping wings with drawn in positions of the respective tip vortices.
To show the generation of vortices also in the area of negative lift this is displayed with very small stages of lift in an enlarged view.
In this case the air flows from top to bottom at the wing tip.
y/s = relative half span
b) The corresponding downwash distributions.

In this figure 8, the change in size of the lift distributions is too strong for birds in my assessment. Perhaps the geometric angle of incidence still may change a little at the wing root[10]. Besides this, the selected distributions generate relatively much thrust. But birds need it only in special flight situations. For the cruise flight of birds one will be able to move the distributions closer to that of gliding flight. Of course, there are also lift displacements without changes in size. But with wing twisting alone it is hardly feasible[10].

Birds certainly are using similar distribution shapes. They also may use special changes of the angle of attack and the airfoils in the elbow area of the arm wing. But about this nothing is known here. Anyway, in flapping flight the position of the tip vortices will be strongly influenced by the shape of the lift distribution.

## 2.4 The slipstream of the flapping wing

The change of the vortex distance from the wing root during a flapping period at the flight of the hooded crow is already noticed in Figure 5. If you look more closely at this picture, you can see in the rear view of the flying bird that the wing tip or the starting point of the tip vortex on the wing performs a revolving or circular motion during one beat period. If one imagines additionally also the forward movement of the bird, we recognize in spatially sighting the helical shape of the rearwards laying wing tip vortex.

Also, the tip vortices of a propeller blade are arranged helically. They enwind the slipstream (or blast of air, thrust jet or thrust stream?) and are an essential part of it. Compared to the propeller, at the flapping wing the windings of the tip vortex are only further expanded and the inner tip vortex lacks. The trajectory of the wing tip or of the wing tip vortex of the bird has therefore at least a strong resemblance with the vortex image of a propeller.

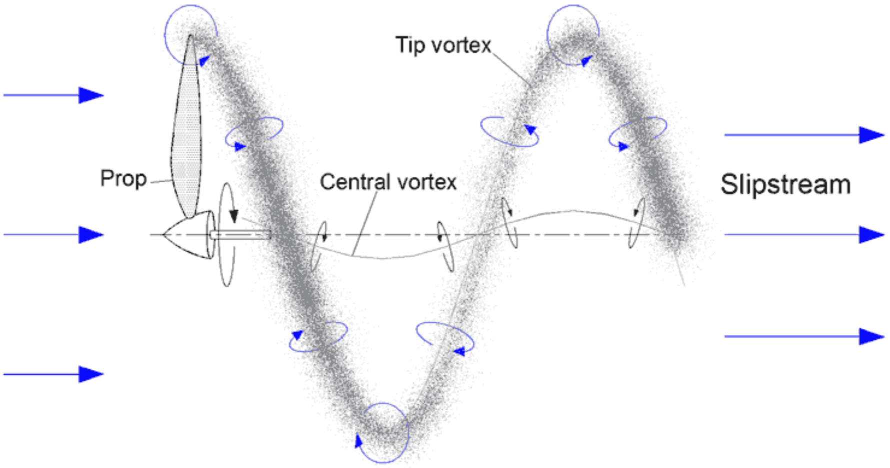

**Figure 9.** Vortex image of a single propeller blade.

Because the lift decreases to zero on both ends of a propeller blade it always has two tip vortices. To provide a better overview the central vortex along the propeller axis only is implied here by a line. But it is as strong as the outer tip vortex.

The propeller blade sweeps during one rotation the surface of the slipstream. In a similar way during the flapping motion sweeps the wing section located between the wing root and the tip vortex an area. But this slipstream area of the flapping wing is changeable.

During a whole flapping period the slipstream area is located between the final stroke positions of the flapping wing and can be represented schematically as follows (see Figure 10). These representations for an ornithopter model are based on lift distributions corresponding to those in Figure 8. But the calculation was made only for one vortex filament with average lift strength. However, it can stand as a good approximation for the whole wing tip vortex. Considering several vortex filaments, the slipstream area gets more circular.

By the wing swept slipstream area near to the fuselage is shared between up and downstroke. On upstroke it is penetrated of the tip vortex in the opposite direction as on downstroke. So the slipstream of this area on upstroke is directed forward. In this way, there generated thrust of the downstroke will be canceled during upstroke (at least at the same vorticity). Only the differential area of up and downstroke remains as slipstream of the flapping wing.

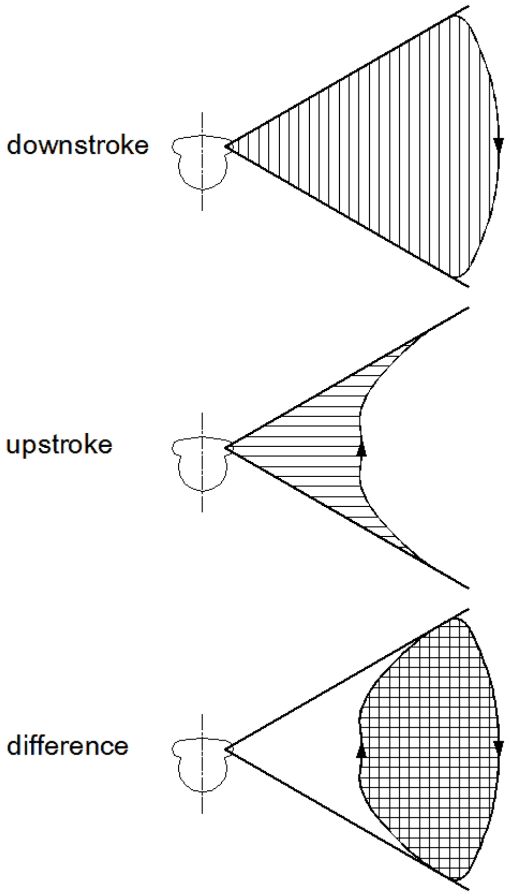

**Figure 10.**
View on a flapping wing in flight direction with the shaded areas that are swept by the wing section between the wing root and tip vortex.
The differential area between up and downstroke equates to the slipstream area of the flapping wing.

Now one could list a number of similarities and differences between the slipstreams of propellers and flapping wings. But certainly in both cases the essential point of thrust strength is the size of the cross-sectional area of the slipstream. On the flapping wing beside the span and the stroke angle in particular the displacement of lift plays an essential role. For a strong thrust, during the downstroke the lift is to displace as far as possible to the outboard wing area. But thereby the induced drag increases very much. On upstroke negative lift in the area of the wing tip supports the attempt to displace the positive lift as far as possible toward the wing root. But the disadvantageous effects on the size and constancy of lift generation must always be considered. What is advantageous for the thrust proves mostly disadvantageous for the lift and vice versa.

The two tip vortices of a whole flapping wing will move down together with the wake located in between. So rearwards, the slipstreams have a slight inclination downwards. But it has to be explored how this wake looks like in detail during the flapping motion and how it effects on the tip vortices. The primary task for the thrust generation during flapping flight is the displacement of lift. At the same time changing arrangement of the tip vortices is only an external sign of this.

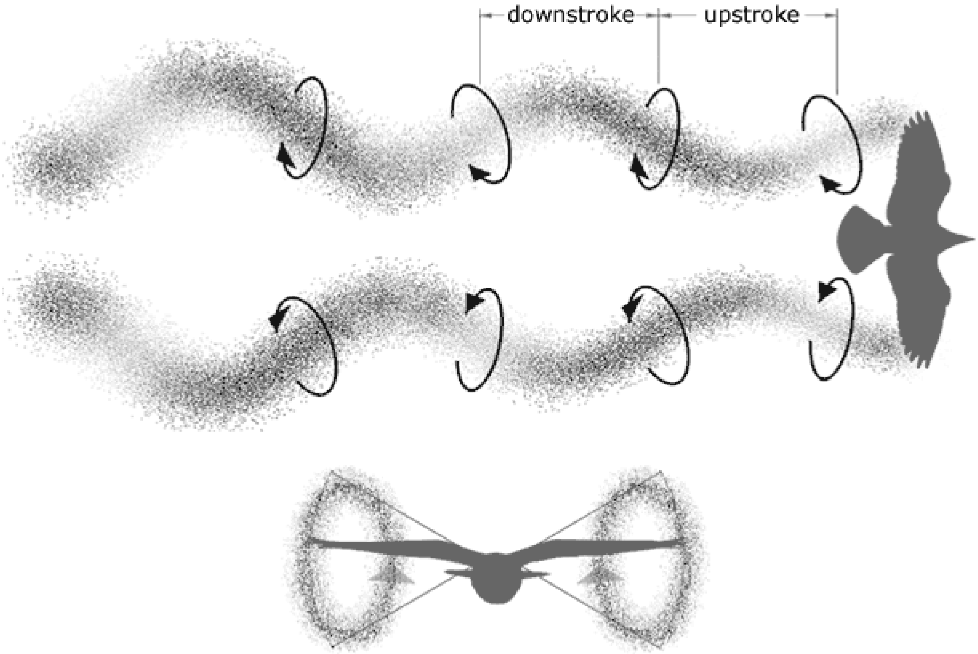

**Figure 11.** The slipstreams of a bird in cruising flight.

For displacement of lift birds have two methods available. They can change the wing shape and thereby also the course of the lift distribution (see Figure 5) or they let the wing stretched and with the course of the angle of attack along the wing they are changing only the lift distribution (see Figure 8). Surely in the course of a wing beat period they nearly always used simultaneously both possibilities. The displacement of the tip vortex has already been observed in birds[11] (like Figure 11).

The ornithopters in the current state of technology have only one possibility to displace the lift distribution. It’s the changing of the lift distribution on outstretched wing by suitable twisting. But there are already approaches for flapping wing designs with a bending motion or also with a sweeping motion to the rear[12] of the hand wing.

## 2.5 Fan effect of the flapping wing along the span

For the mode of operation of the slipstream it is also important to considered transition between up- and downstroke. Thereto, A. Piskorsch[13] has shown me slow-motion movies of swans and storks, which were taken in the line of sight from the front or rear. Thereby the following impression has formally influenced me.

The motion of the flapping wing runs, expressed somewhat exaggerated, such as a whip, whose ability to bend upwards is limited. The upstroke starts with the lifting of the arm wing. At the same time takes place a downward directed bending of the hand wing and the lightly loaded pinion feathers point downward. They give the hand wing a weakly curved shape. During the upstroke the bending runs, at least seemingly, like a wave from the wing root towards the wing tip. The smaller wing bending of the downstroke blends harmoniously on it. The wave motion along the span already ends a short time after the beginning of the downstroke. There already exists the maximum deflection of the wing and the pinion feathers. This wing attitude is kept for most of the whole downstroke. In the lower final wing position the wave restarts from the beginning at the wing root. In particular, during the starting process one can literally imagine a packet of air under the wing which is transmitted by the wave from the inside to the outside. One can also simulate the whole process in front of you by imitating it with your own hands and arms.

The effect of this whip-shaped movement process can be compared with that of a fish tail, or roughly seen with that of an air fan. Hence, the flapping wing accelerates the air not only rearward and downward but also to the side. In birds, also the geometric angle of incidence along the span does not change so abruptly as in ~~the~~ most ornithopters with membrane wings. Instead, the variation of the angle of incidence runs along the span like a wave together with the bending of the wing.

The temporal and local variation of the change in the angle of incidence on upstroke can be recognized well on hand of images in the film of a flying swan[14]. They were taken exactly from the front. The upstroke begins with the shaping of a large angle of incidence at the wing root. The hand wing takes part in them only markedly delayed. Only during the movement as from the middle of the upstroke the wing twisting takes the shape as one generally expected on upstroke. The strong lift of the downstroke is obviously displaced at the beginning of the upstroke, in almost stretched wing, from the hand wing towards the wing root.

In the upper final wing position, displacement takes place in the opposite direction. Once the arm wing has reached the upper final position it takes about the angel of incidence of the downstroke. With the strong camber of its air foil thereby it certainly assumed much of the lift. In this position the arm wing is waiting until the hand wing has finished its upstroke. This waiting is absent in the previous ornithopters with bent wings. First together with the remaining upstroke motion of the hand wing it’s built up the greater lift in the outer wing area. It will be observable on the deflection of the pinions on the wing tip after the ending of the upstroke motion or at the beginning of the downstroke motion. Thus already in the area of the upper end position the lift respectively its tip vortex is displaced from inside to the outside. This is good for the size of the slipstream cross section.

One also can see the effect of the fan in the picture of the displacement of the tip vortex of a bird[11] (see as well as Figure 11). The distance between the two tip vortices behind the flying bird expands more and more, in particular at the downstroke. So the center line of the slipstream points a little outwards.

The spread of the tip vortex behind the wing corresponds to an apparent increasing of the span. Thereby the induced drag decreases, and further behind the double vortex of the whole flapping wing itself may roll together.

## 2.6 Starting vortices

At the transition between the both stroke cycles it would be good if both cycles are working with approximately the same size of lift. Otherwise, with every new increasing lift it leads to a starting vortex (see „Die aerodynamischen Grundlagen des Schwingenfluges“[15] by A. Lippisch, 1938). It is blown rearwards after its emergence, but remains connected to the wings over free vortices. During reduction of the lift then also these parts of the vortex loop lags (see Figure 12). The generation of the starting vortex requires energy again and again. An equalisation of the lift size of both stroke cycles leads not only to smaller up and down motions of the model but also to saving of energy.

The benefits of a good transition between the two stroke cycles however, are not recognizable in a calculation method with quasi-stationary flow conditions. Under stationary conditions, there are no starting vortices.

The energy for the starting vortex can be estimated. Thereto first you set its length into relation with the length of the rest, horseshoe-shaped vortex. Thereby equates the length of the horseshoe vortex to the energy which is converted with the induced drag.

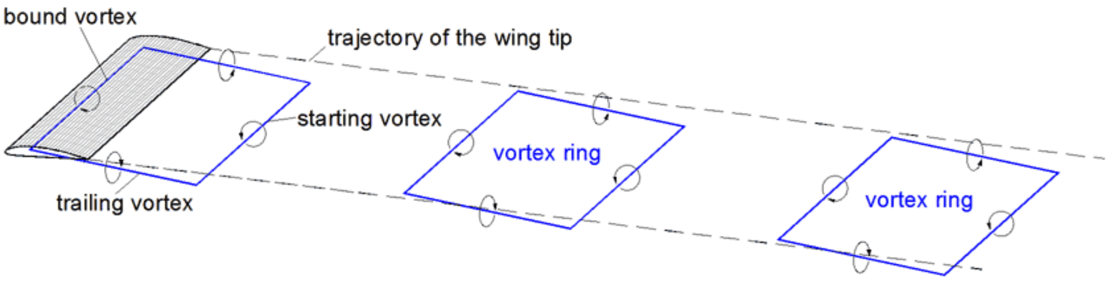

**Figure 12.** At its point of origin frozen vortex rings along the flight path, with cyclically changing lift. When unfreezing the vortex rings moved downwards.
The length of the vortex rings and their distance to each other is based to the wingspan significantly reduced.

The starting vortex is short compared with the horseshoe vortex. In addition, only the lift difference between the two stroke cycles is decisive for the intensity of the starting vortex. Overall, therefore, the energy consumption for the starting vortex is relatively low. Nevertheless, on ornithopters one should try, at least approximately, to equalize the lift size of two stroke cycles.

# 3 Utilization of the vortex system in formation flight

## 3.1 Use of the upwash

On a wing there are not only the two wing tip vortices and downwash in between, but also a movement of air upwards around the wing tip. Directly at the wing tip there is therefore an upwash. A popular thesis of the formation flight of birds assumes that this upwash of the flying leading bird can be used by the trailing bird. This imagination is usually explained only by the vortices on the wing tips during gliding flight. But the whole vortex system behind the wing will in practice never be considered.

**Figure 13.** View on a flying goose and the upwash left and right of it.

From the air flow around the wing tips behind the wing caused the wing tip vortices. They are rolling downward together with the downwash velocity $w_i$ through the surrounding air space. Together they formed a double vortex. In it is included the whole air mass captured from the wing (Figure 14). Because its broadness it limits the size of the single tip vortices. On large aircrafts wing tip vortices reached diameters approximately to those of the span[16]. The downward motion of the double vortex complies also with the rules of the lift theory by Newton.

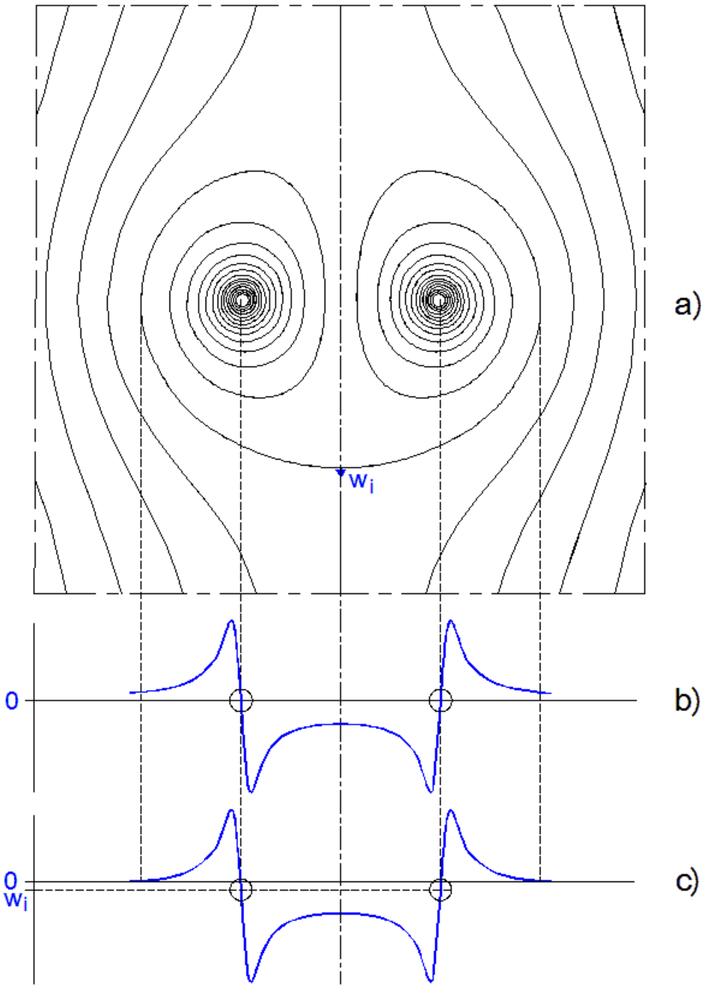

**Figure 14.** a) Cross section of the double vortex of an elliptical lift distribution at some distance behind the wing.
b) Downwash velocity $w_i$ based on the double vortex.
c) Downwash velocity $w_i$ based on the surrounding air space.

Each of the two tip vortices rolls on its outside on surrounding airspace downwards. Behind aircrafts which are flying through clouds can see only a cloud gorge in span width but besides no cloud mountains. Where now, the trailing bird shall go to benefit from an upwash of his leading bird?

In the lift thesis for the formation flight it is assumed that the trailing bird lay with a wing tip about in the center of one trailing vortex of his leading bird. He is then with the whole respective wing in the downward moving double vortex of the leading bird.

Nevertheless, the trailing bird has clearly upwash in the outer wing area (see Figure 15). Close to the center of the vortex, however, the flow direction is changing significantly only a few centimeters above or below the vortex axis.

So the trailing bird is raised on one side and thus pushed aside. To compensate, he must countersteer. On upstroke, depending on the lift distribution or thrust requirement, the direction of downwash on the wing tip also reversed (e. g. like in Figure 8b). In what way overall can be reached benefits with unilateral lifting, is not easily discernible.

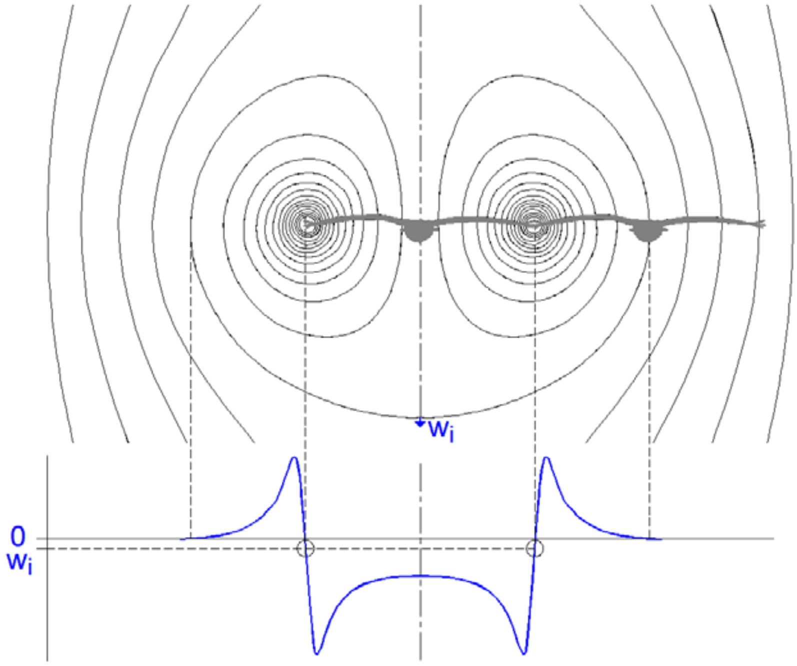

**Figure 15.** Formation flight during gliding flight in upwash of the leading bird (left). Under it is shown the course of the downwash velocity of the leading bird based on the surrounding air space.
The natural tendency for the trailing vortex pair to move toward one another is neglected here (distance = 0.78 x wing span, see Figure 8a, gliding).

Also other arrangement of the trailing bird to the leading bird doesn't lead anywhere in a simple double vortex. That's probably may be the reason why birds don’t fly in a formation on gliding over long distances. Thus, the formation flight should be explained by characteristics of the flapping flight.

## 3.2 Drag reduction

Another thesis about the formation flight is based on the idea that the trailing bird can reduce the strength of its own tip vortex. He only must place the starting point of his own wing tip vortex in the tip vortex coming into him from the leading bird. Because of the opposite direction of rotation of both tip vortices the induced drag can be reduced on the trailing bird (see following Figure 16).

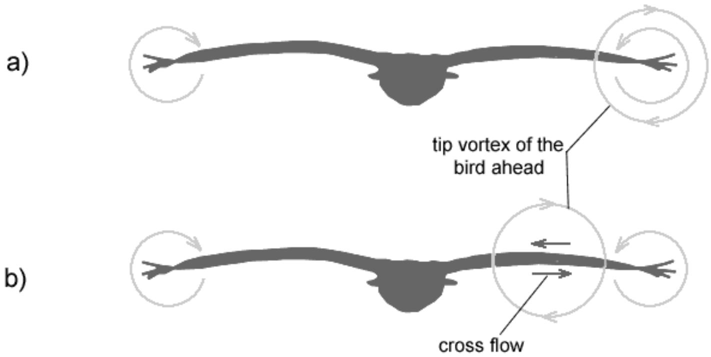

**Figure 16.** a) Counter-rotating tip vortices in a formation flight to reduce the induced drag.
b) Reduction of the cross flow on a wing, by utilization of the tip vortex from the leading bird.

This method has the advantage that the lateral distance does not have a very great significance. The effect is even then effective if the tip vortex of the leading bird is located in the wing area, but only nearly the starting point of his own wing tip vortex. In this case the cross-flow on the wing and thus the induced resistance become smaller. However, the latter arrangement is not advisable. Then the outside wing area of the trailing bird lies in the downwash of the leading bird, at least during its wing downstroke (see Figure 15).

In gliding flight here the arrangement of the two birds is the same as in the lift thesis. So the trailing birds feel the unilateral lift, simultaneously the induced drag on this wing side get smaller. For the straight flight the trailing bird must permanently counteracting. What's the advantage? Also this method is not used by the birds in the gliding flight.

When flapping flight seen in direction of flight each wing tip vortex performs of a circular movement (see Figure 5 and 10). The middle trajectory and therefore also the cross-section of the slipstream have roughly the shape of an ellipse. For the trailing bird it is now possible be bring its own slipstream into alignment to the slipstream of the leading bird (see following Figure 17).

But the trajectories of the two tip vortices which are forming then the combined slipstream are not congruent. As seen in Figure 5 and 10, they circuits the combined slipstream in opposite directions. They are crossing only twice in a flapping period. When flapping flight, hence it is not available to place the tip vortices of the leading and trailing bird superimposable about each other. So one cannot benefits from the counter-rotating tip vortices.

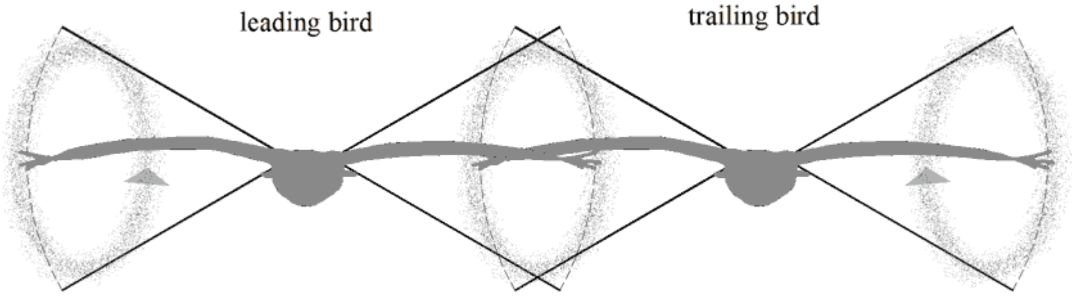

**Figure 17.** Arrangement of the birds with a united slipstream (middle tip vortices)

Whether the series connection of the two slipstreams, similar to the case of two counter-rotating propellers, offers appreciable advantages is unknown to me. In particular, thereby the rear propeller should function better. That would be then a further thesis. The arrangement of the birds in Figure 17 in any case can be seen relatively common in V-formations of birds.

## 3.3 Ride on the slipstream

Together with the slipstream theory of a flapping wing there is here another thesis for the function mechanism of the formation flight. The trailing bird could also locate his body centrally above the slipstream. On whose outside there is a forward directed flow (see Figure 9). This could be used by the trailing bird as a tailwind. But then, the trailing bird could perhaps lies in the area of the downward moving double vortex of the leading bird. In addition, the contact area between the vortex and the trailing bird is very small and only short-time existing.

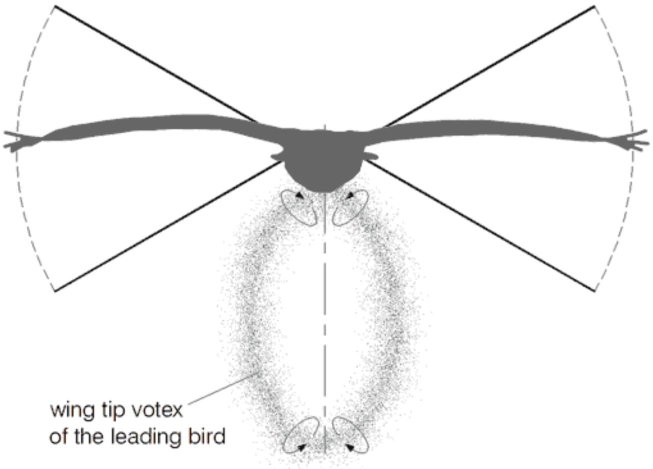

**Figure 18.** Ride on the slipstream (view from behind).

When checking all these theses on basis of images of the formation flight of birds is taken into account that the center line of the tip vortices and the slipstream is not exactly in the direction of flight. Behind the bird they are directed a little bit towards the outside. But certainly, the angle is different depending on the bird species.

Birds might use some of the options that are described here, depending on level flight, climb flight or fast flight. Also, each bird species might prefer and optimize another version. On images from formation flight anyway always fit just a few birds of a formation in one of the schemes described here. Perhaps, one could harden or exclude one or the other thesis, if one had not only vertical but also lateral images of the same V-formation. Then one could at least estimate in which high the trailing bird is flying through the tip vortex of his leading bird. Anyway, many questions are open on the theme of "wing tip vortices of flapping wings".

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