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The interesting article by Archer, Sapuppo, and Betteridge[1] leads to the problem of determining the optimum lift distribution along the wing during flapping motion. Adopting the assumptions made in Ref. 1, the problem may be reduced to one of minimizing the induced drag for a specified, periodically varying, bending moment at the wing root. The wing-root bending moment should be of primary importance to a bird or, for that matter, to a mechanical flapping machine, since it is at this point that most of the flapping energy is introduced.

To solve this problem we adapt a technique employed by Prandtl and Munk. Assume at first that no net lift is specified, that is, the wing is used solely for propulsion. We seek to determine the load distribution $\ell(y)$ that minimizes the vortex drag D_i with a given bending moment M. Assume that $\ell(y)$ is given. Then small variations in the shape, $\delta\ell(y)$, will produce no first-order change in D_i or M. Since two conditions are involved, it is sufficient to consider two discrete elements of variation (see Fig. 1). The added drag will have three components.

(1) The drag of the added distribution $\delta\ell_1, \delta\ell_2$ in the downwash w_i of the original load distribution $\ell(y)$.

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(2) The drag of the original distribution in the downwash of the added distribution.

(3) The drag of the added distribution alone.

According to Munk's reciprocal theorem (the "mutual drag" theorem), items (1) and (2) are equal; hence we need consider only item (1). Item (3) is proportional to the square of the added lift and hence does not contribute to the first-order variation. We then have the equations:

$$\delta\ell_1 y_1 + \delta\ell_2 y_2 = 0 \text{ (for } \delta M = 0) \tag{1}$$

$$\delta\ell_1 w_{i_1} + \delta\ell_2 w_{i_2} = 0 \text{ (for } \delta D_i = 0) \tag{2}$$

These equations will be satisfied for all positions of $\delta\ell_1, \delta\ell_2$ if we make

$$w_i = \omega_i y \quad , \tag{3}$$

where ω_i is a constant. Thus, we see that for minimum drag in flapping motion the induced downwash should vary linearly along the span; that is, the vortex trace of the wing should move as a rigid surface hinged at the wing root.

In Munk's original problem, the total lift and the wing span were given. There it was found that for minimum drag the induced downwash should be constant along the span or, in other words, the vortex trace of the wing should move downward as a rigid surface. This holds as well for wings having more complex shapes in front view, such as gull wings or wings with "winglets." Betz even extended this idea to propellers, showing that for minimum energy loss the vortex trace of a propeller should move backward as a rigid helix; the potential flow solution for this motion was found by Goldstein. That such ideas should persist in the case of flapping motion is therefore not entirely unexpected.

The extension of eqn. (3) to more complex shapes, such as gull wings, is easily accomplished by introducing the appropriate cosines into eqns. (1) and (2). In such cases it is found that minimum energy loss or drag occurs when the wake moves so as to satisfy the boundary condition of an impermeable surface having the shape of the wing trace and executing a similar motion.

In eqn. (2) we assumed that the vortex drag could be calculated by considering an induced downwash w_i at the wing. This assumption limits our calculation to wings of high aspect ratio and to cases in which the wave length of the flapping motion is long compared with the wind chord. A still simpler situation arises if we consider very slow flapping such that the wavelength is large compared with the wing semispan (as assumed in Ref. 1). In this case, the motion of the wake is approximately two-dimensional and conventional induced-drag theory can be used.

Adopting the latter assumption, we have to find the spanwise load distribution that corresponds to a linear variation of the induced downwash w_i/V. Using the standard inversion formula of airfoil theory we obtain (see Ref. 2)

$$\Gamma = \frac{4}{\pi}\,\omega_i s^2\left(\sqrt{1 - \frac{y^2}{s^2}} + \frac{y^2}{s^2}\cosh^{-1}\frac{s}{|y|}\right) \qquad (4)$$

for the ideal distribution of circulation Γ during slow flapping (see Fig. 2). Here s is the wing semispan; it will be seen from eqn. (3)

that the quantity $\omega_i s/V$ is the downwash angle v_i/V at the wing tip. If ΔC_L is the additional lift coefficient during flapping we obtain

$$\frac{\omega_i s}{V} = \frac{3}{4AR} \Delta C_L \quad . \tag{5}$$

If we let ω be the angular velocity of the flapping motion then ωy will be the vertical velocity of the motion at y. The instantaneous thrust will be

$$T = 2\int_0^s \rho\Gamma(\omega - \omega_i)y \, dy \quad . \tag{6}$$

Averaging over a cycle and forming the coefficient

$$C_T = \frac{T}{S \frac{\rho}{2} V^2} \quad , \tag{7}$$

we find

$$C_T = \frac{1}{\pi} \frac{(\omega - \omega_i)s}{V} \frac{\omega_i s}{V} AR \quad , \tag{8}$$

where ω and ω_i are now maximum values. The efficiency is

$$\eta = \frac{TV}{2M\omega} = 1 - \frac{\omega_i}{\omega} \tag{9}$$

where M is the bending moment. The loss of efficiency is thus simply the ratio of the angular velocity induced by the wake to the angular velocity of the wing. At its maximum the efficiency is constant along the span. Introducing eqn. (9) into (8) we find

$$\frac{C_T}{AR} = \frac{1}{\pi} \eta(1 - \eta)\left(\frac{\omega s}{V}\right)^2 \quad , \tag{10}$$

which is a simple equation connecting the thrust, the efficiency, and the tip helix angle. Figure 3 shows the relation between thrust and efficiency. Figure 4 shows the efficiency given by the optimum loading (eqn. (4)) compared with that determined in Ref. 3 for an elliptic loading. It will be noted that for a given value of thrust there are two possible values of the efficiency, the lower value evidently corresponding to an excessively high lift coefficient.

Expressing ω_i in terms of the additional lift (eqn. (5)) we find

$$\eta = 1 - \frac{3}{4\mathit{AR}} \frac{\Delta C_L}{\omega s/V} \quad , \tag{11}$$

which shows the influence of aspect ratio. Integration over a periodic cycle shows that the efficiency is a maximum when the incremental lift ΔC_L is in phase with the upward or downward velocity of the wing tip. Hence, ΔC_L and ω in eqn. (10) may be maximum values during the cycle. However, for best efficiency the lift increment should be small and the vertical velocity of the wing tip large—leading to rather large angles of twist during flapping, as found in Ref. 1.

The exact angle of twist required to obtain the optimum loading will depend on planform of the wing. To compute this we make use of the formulas

$$\Gamma = \frac{1}{2} C_l \, Vc \quad , \tag{12}$$

where C_l is now the local lift coefficient and c the local wing chord. In addition we have

$$C_l = 2\pi \left(\frac{\omega y}{V} - \frac{\omega_i y}{V} + \Delta\alpha_g \right) \quad , \tag{13}$$

where $\Delta\alpha_g$ is the geometric twist angle introduced during the flapping.

Figure 5 shows curves of $\alpha_g/(\omega s/V)$ for elliptic wings of $AR = 6$ and 10. It appears that the optimum twist for the elliptic wing is almost linear, conforming to the flapping motion. However, a small additional positive angle of attack appears to be needed at the wing root. This variation will, of course, be influenced by the planform shape of the wing.

A complete analysis of the motion of a bird would involve vertical motions of the body in addition to the pure flapping motion considered here. Such vertical motions will not only influence the optimum twist distribution, but will also contribute directly to the propulsion—possibly in a negative sense.

Since the flapping wing is also normally used for support, it is necessary to consider the possible "interference drag" between the steady, supporting lift and the alternating lift due to flapping. According to Munk's reciprocal drag theorem we need to consider only one term of the interference drag. Taking the drag of the steady distribution in the downwash field of alternating distribution, it is easy to see that the interference is zero when averaged over a cycle, since the downwash of the added distribution is alternately positive and negative.

It is of interest to compare the values of thrust given in Fig. 3 with those required by a bird for steady flight. To maintain level flight we must have

$$C_T = C_D \tag{14}$$

The drag coefficient of a well streamlined large bird might be as low as 0.025. Assuming an aspect ratio of 8,

$$\frac{10\ C_T}{Æ} \cong 0.03 \quad . \tag{15}$$

For a tip velocity ratio of 0.3, the propulsive efficiency is about 87%.

It has been suggested that the optimum load distribution during steady (nonflapping) flight would be the elliptic loading. However, as shown in Ref. 2, a 15% reduction of induced drag can be achieved for the same bending moment by adopting a more tapered form for the steady component of the load distribution. Although the variety seen in the wings of birds discourages generalization, this fact may explain why long-flying species, such as the gull and the Frigate bird, have more tapered wings.

The foregoing analysis is easily extended to the case of a propeller operating at a large advance angle. In this case, we are interested in the rolling moment or torque rather than the bending moment of the wing root. Equations (1) and (2) remain the same but the "downwash" given by eqn. (3) is now antisymmetric instead of symmetric and the absolute value sign on y is removed. The wake then revolves as a rigid helical strip, in agreement with the Betz-Goldstein result.

The optimum spanwise loading in this case is simply

$$\Gamma = 4\ \omega_1 s^2 \left(\frac{y}{s}\sqrt{1 - \frac{y^2}{s^2}}\right) \quad , \tag{16}$$

giving the thrust coefficient,

$$C_T = \frac{T}{s\,\rho/2\,V^2} = \frac{\pi}{8}\,Æ\left(\frac{\omega s}{V}\right)^2 \eta(1-\eta) \quad , \tag{17}$$

where η is the efficiency. As in the case of flapping, the induced loss is the ratio of half the angular velocity of the far wake to the angular velocity of the wing or blade.

Under the assumptions of slow flapping, or slow turning of the propeller, the normal slipstream loss considered in the actuator disc theory (Froude theory) will be negligible. The primary loss is the "rotational inflow" considered here. As is well known, this loss may be substantially overcome by using counterrotating propellers in tandem.

The principle of the counterrotating propeller could be extended to wing flapping—a mode that might be termed "counterflapping." Two wings in a close tandem arrangement and moving in opposite phase would eliminate the induced losses we have calculated here. It is interesting that this mode of propulsion does not appear in nature—except possibly in the dragonfly (see Ref. 4).

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3. BETTERIDGE, D. S. and ARCHER, R. D. A study of the mechanics of flapping wings. Aeronautical Quarterly, May 1974.

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FIGURE CAPTIONS

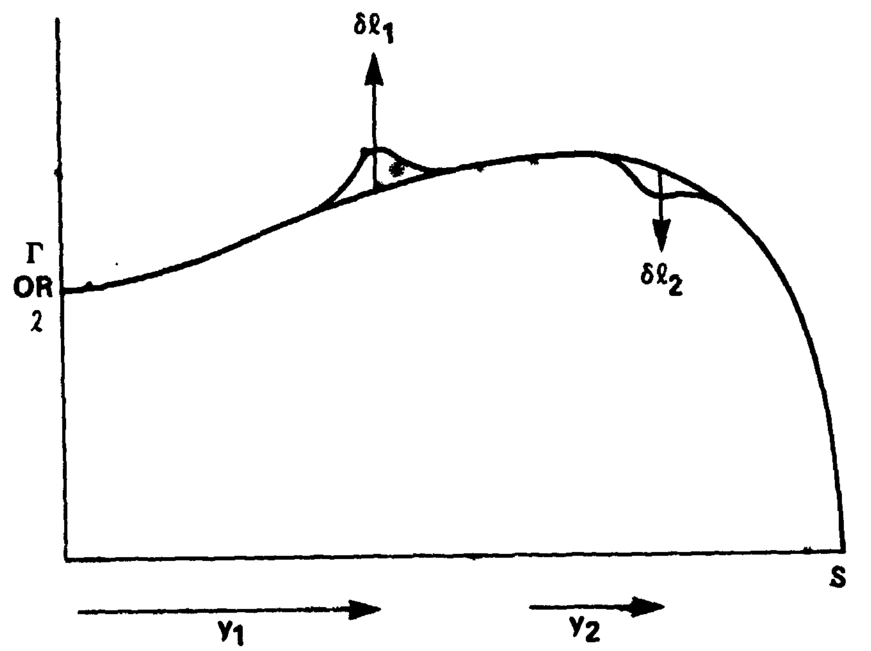
Figure 1. Variational problem for flapping with minimum energy.

Figure 2. Ideal distribution of the additional lift during slow flapping.

Figure 3. Thrust and efficiency at different values of the tip velocity ratio.

Figure 4. Elliptic loading and optimum loading compared with $\omega s/V = 0.3$.

Figure 5. Angle of twist during flapping elliptic planform: $\eta = 80\%$.



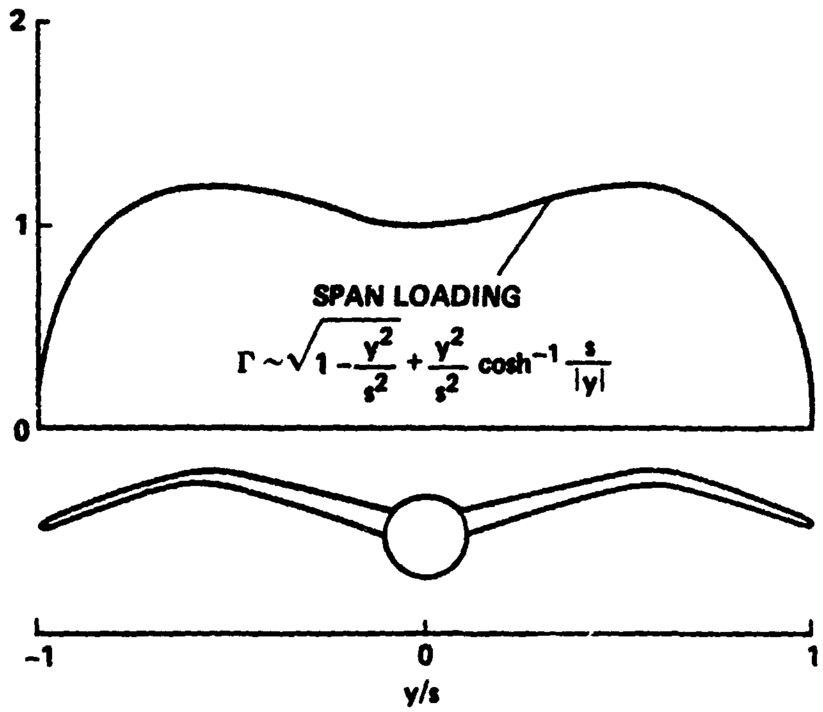
$$\delta\ell_1 Y_1 + \delta\ell_2 Y_2 = 0$$

$$\delta\ell_1 \underline{w_1} + \delta\ell_2 \underline{w_2} = 0$$

HENCE

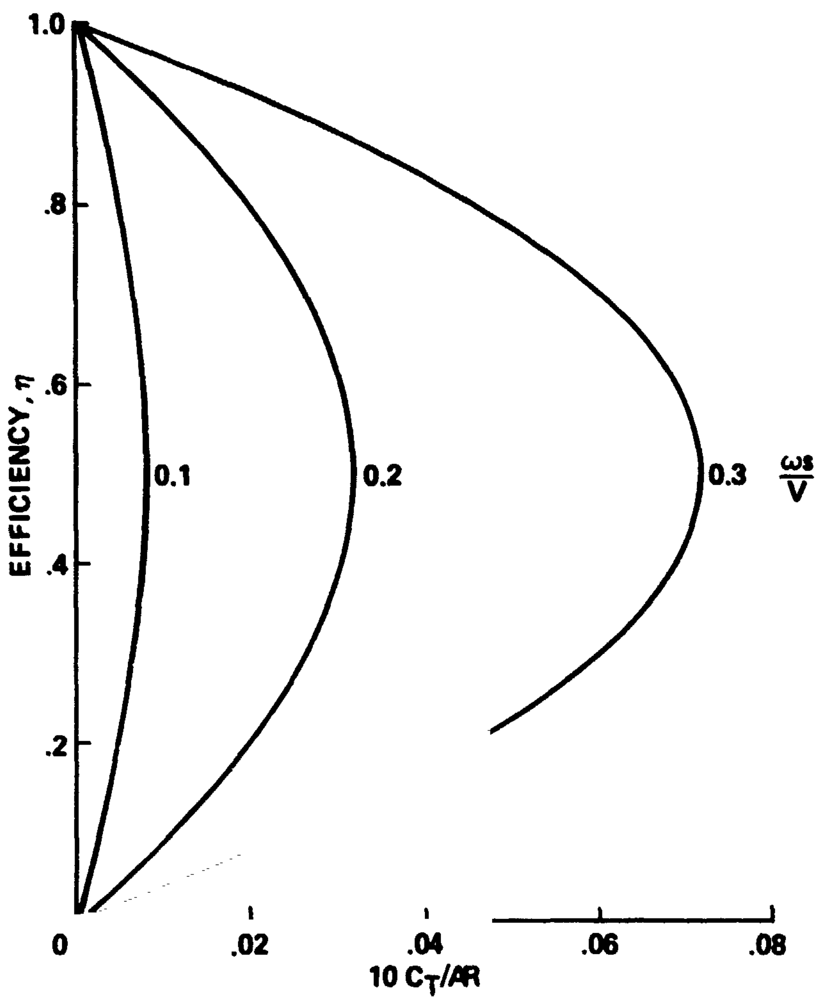
$$w_i = \omega_i Y$$

Fig. 1



2
1
0
SPAN LOADING
$\Gamma \sim \sqrt{1-\frac{y^2}{s^2}} + \frac{y^2}{s^2}\cosh^{-1}\frac{s}{|y|}$
-1
0
1
y/s

Fig. 2



1.0
.8
.6
.4
.2
0
EFFICIENCY, η
0.1
0.2
0.3
$\frac{\omega s}{V}$
.02
.04
.06
.08
10 C_T/AR

Fig. 3

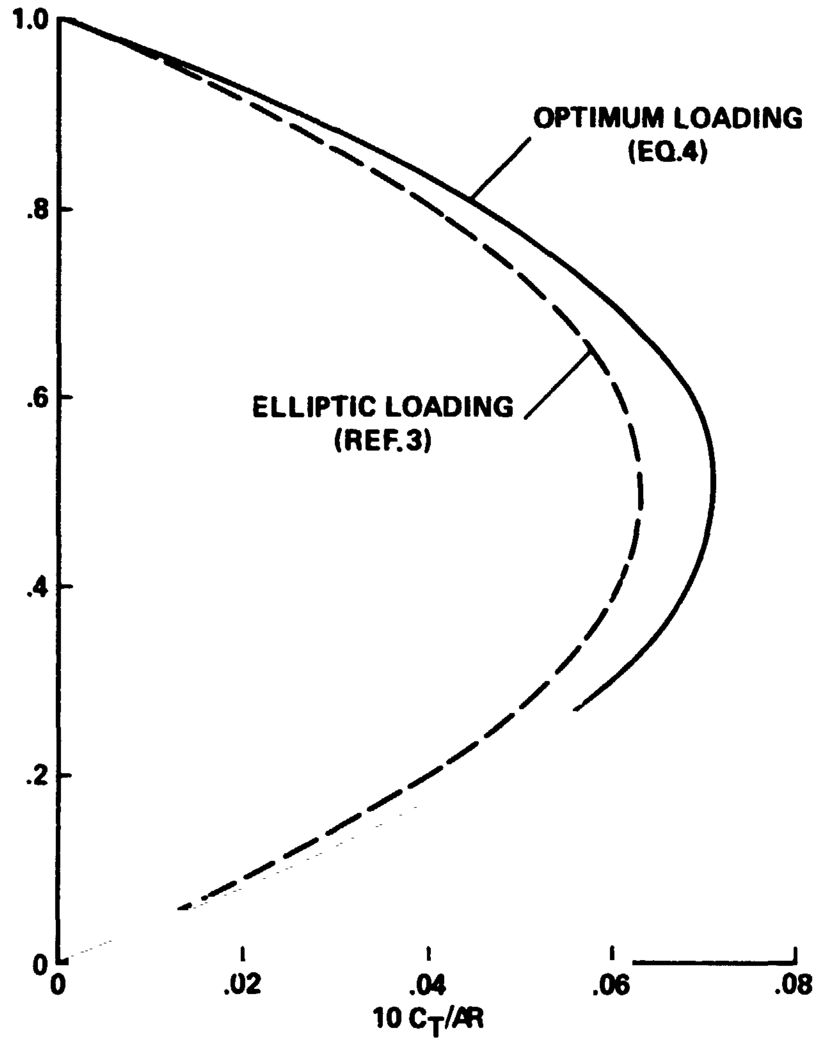

1.0
.8
.6
.4
.2
0
0
.02
.04
.06
.08
10 C_T/AR
OPTIMUM LOADING
(EQ.4)
ELLIPTIC LOADING
(REF.3)

Fig. 4

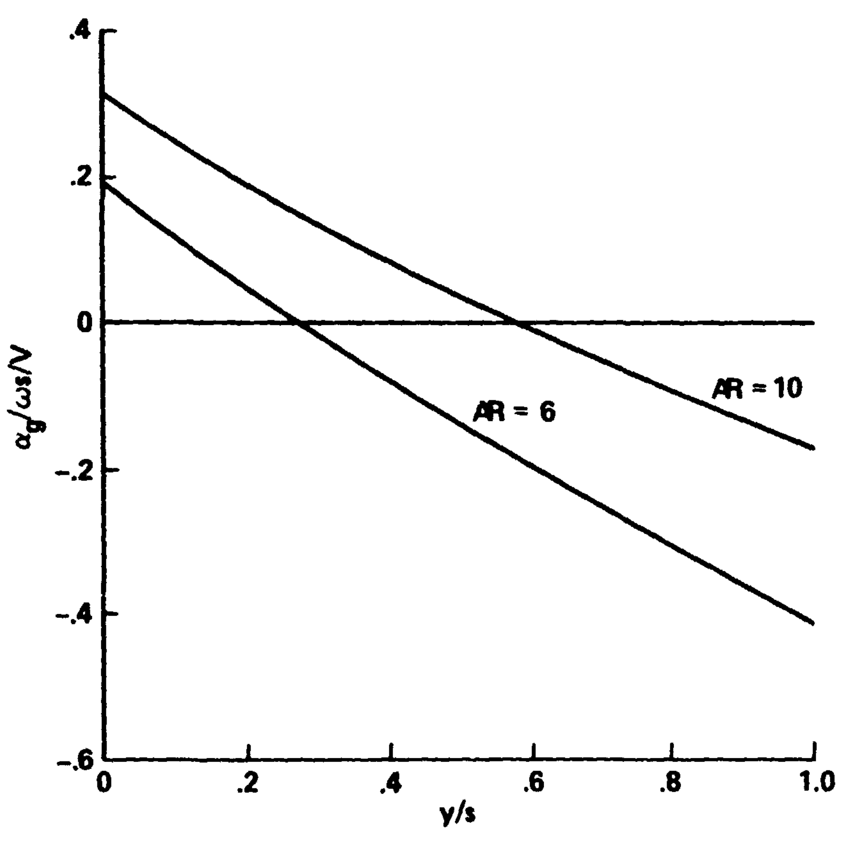


Fig. 5